Ideal mode expansion for planar optical waveguides: application to the TM–TM coupling coefficient for grating structures

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We derive the coupled equations for the ideal modes of an optical waveguide, using a Green function technique, and use these to determine the TM–TM coupling coefficient for a periodic waveguide diffraction grating. The results are consistent with experimental observations, in marked contrast to the results of more conventional versions of the ideal mode expansion. Further, our approach can deal with more general corrugated structures than can easily be done with other methods.

1. INTRODUCTION

The study of the influence of imperfections on the electromagnetic properties of otherwise ideal waveguides is critical for a complete understanding of integrated-optical systems and devices. These imperfections either can be unintentional, owing for example, to incomplete control over the fabrication process, or can be intentionally fabricated, for example, in grated waveguide devices. Imperfections will in general change the propagation constants of the electromagnetic modes of the system and can, in addition, induce a coupling between the different modes. To analyze these effects several methods, including normal mode analyses,1–7 total field calculations,8 and a generalized ray optics method,9 have been used, but not all these methods lead to identical results in all cases. A notorious example of this is the TM–TM coupling coefficient for a waveguide diffraction grating, at both normal and oblique angles of incidence. The discrepancies among the various methods, which are well documented,7,8,10 are associated with the application of the boundary conditions at the interface of the guiding layer and the cover, as recently discussed by Weller-Brophy and Hall.7 These authors show that the total field analysis8 and the local normal mode expansions2,7 lead to identical results, which differ from results based on ideal mode expansions,3,5,8 which, in turn, differ from the results of a modified ideal mode expansion proposed by Streifer et al.4 A measurement of the TM–TM coupling coefficient as a function of incident angle by Weller-Brophy and Hall11 is in clear contradiction to the results of ideal mode expansions but cannot distinguish between the local normal mode2,7 and the total field methods on the one hand and Streifer’s method on the other.4

In the present paper we use an alternative way to derive the equations for the interactions between the normal modes of the ideal waveguide, while avoiding the problems associated with earlier treatments.10 We can use this newly derived ideal mode expansion, which makes use of the Green function of the ideal system, to evaluate the TM–TM coupling coefficient for a periodic waveguide grating and find that the result is in perfect agreement with those of the local normal mode expansion and the total field analysis but differs from the results of Streifer’s method. Our method, which constitutes an independent way to calculate coupling coefficients, thus clearly produces results with only one of the existing results. Apart from this, our method has important advantages when applied to waveguide geometries in which an additional refractive index is introduced. The analysis of such geometries, in which there has been some recent interest,12 poses no particular problem when an ideal mode expansion is used but can be tedious when the other methods are used. In addition, our method has the attractive features that it quite naturally takes finite beam effects into account and that it is well suited to include nonharmonic electromagnetic fields. The latter feature is particularly convenient when one is studying pulse propagation through nonlinear waveguides. The present analysis holds only for shallow imperfections, which means that that the actual waveguide does not differ severely from the ideal. This restriction limits the modulation of the thickness of the guiding layer in a grating structure to ∼λ/10, where λ is the wavelength of the radiation in vacuum. Iterative methods can be employed to describe the properties of deeper gratings,9 but we will not do so here. Naturally, in the course of our derivation we find an explicit expression for the Green function of the ideal waveguide. It can provide a convenient starting point in investigations for which our subsequent approximations, like that of a shallow grating, are not valid.

The present work represents a significant extension of earlier work by one of us and a coworker in which the Green function for one-dimensional waveguide problems was derived.13 These results are briefly rederived in Section 2 and are generalized in Section 3 to find the general Green function for an ideal planar optical waveguide system. Application of a principal pole approximation, in which the Green function is approximated by neglecting most poles except the few that are of direct interest, enables us in Section 4 to simplify the formalism considerably, which, in turn, allows us to derive the coupled-mode equations and general expressions for the coupling coefficients in Section 5. We evaluate
these expressions for the specialized case of waveguide gratings and then briefly summarize our conclusions.

2. GREEN’S FUNCTION FOR ONE-DIMENSIONAL SOURCES

In this section we derive an expression for the Green function of an ideal waveguide. In this derivation we assume that we have a known source polarization perturbing the waveguide, and the Green function then describes the response of the system to this perturbation. Only in Section 5 will we make the connection between the source polarization on the one hand and the waveguide imperfections on the other. We consider a system as in Fig. 1 and thus choose the guiding layer to be in the y-z plane, so that the x axis is perpendicular to the interfaces of the ideal waveguide. For simplicity, Fig. 1 shows an elementary waveguide structure with only one film material (refractive index $n_f$, relative dielectric constant $\epsilon_f = n_f^2$) bounded by a cladding material (index $n_c$, relative dielectric constant $\epsilon_c = n_c^2$) and a substrate material (index $n_s$, relative dielectric constant $\epsilon_s = n_s^2$). All the results presented in the present paper, however, can be easily generalized to more complicated multilayer ideal waveguide structures.

Here we consider source polarizations that do not depend on the y coordinate. We find the particular solutions to Maxwell’s equations for such geometries and denote these solutions the generated fields. Since the source polarizations are y independent, the generated fields depend only on $x$ and $z$ as well. This is not necessarily true, however, for homogeneous solutions to Maxwell’s equations, so that the total fields, in general, depend on all three coordinates. Here and in Section 3, where we consider a more general source polarization, we write the fields $f(x, t)$ as

$$f(x, t) = f(x)e^{-i\omega t} + \text{c.c.}, \quad (2.1)$$

where c.c. denotes complex conjugation, so the two relevant Maxwell equations attain the form

$$\nabla \times E(r) - i\omega \mu_0 H(r) = 0, \quad (2.2a)$$
$$\nabla \times H(r) + i\omega \epsilon_0 E(r) = -i\omega P(r), \quad (2.2b)$$

where SI units are used and the waveguide is assumed to be nonmagnetic. The dielectric function $\epsilon(x)$ describes the ideal waveguide, changing in a stepwise manner as $x$ crosses from one material to another. As mentioned, $P$ is ultimately to be associated with the imperfections; how this is to be done will be discussed below. Our task for now is to find, subject to the Maxwell saltus conditions and the appropriate boundary conditions at infinity, the electric and magnetic fields generated by a specified polarization. In order to do so, we first separate the longitudinal electric field component, which points in the $z$ direction and is to be denoted by the subscript $l$, $E_l = E \cdot \hat{z}$, from the transverse components $E_\alpha = E \cdot (\hat{x} \pm \hat{y})$; we use the same decomposition for $H$. We then write the generated fields conveniently as

$$F(r) = \begin{bmatrix} E_l(x) \\ H_l(x) \\ E_\alpha(x) \\ H_\alpha(x) \end{bmatrix}, \quad (2.3)$$

Below, we express these fields in terms of the normal modes of the ideal waveguide. The modes, which can travel either toward $z = +\infty$ [in the forward (+) direction] or toward $z = -\infty$ [in the backward (−) direction], are either TE ($\epsilon$) or TM ($\mu$) polarized and can be written as

$$F^\pm(r) = a_\pm \begin{bmatrix} \pm \epsilon_\alpha^{\pm}(x) \\ \pm e_\alpha^{\pm}(x) \\ 0 \end{bmatrix} \exp(\pm i\beta z), \quad (2.4)$$

where $F^\pm(r)$ are the normal modes with amplitudes, and

$$e_\alpha^{\pm}(x) = e_\alpha^{\pm}(x)\hat{y}, \quad h_\alpha^{\pm}(x) = h_\alpha^{\pm}(x)\hat{x}, \quad (2.5)$$

Note that Eqs. (2.4) contain an implicit convention regarding the field directions associated with forward- and backward-propagating modes of equal amplitude. Two such TE modes, for example, have opposite electric fields, whereas for the more common convention in Refs. 2, 3, and 7 two such modes have identical electric fields. To describe the same physical phenomenon, for example, Bragg reflection off a periodic surface corrugation, these two conventions therefore lead to opposite amplitudes of the backward-scattered wave. Since the coupling coefficient gives the relation between the amplitude of the forward- and backward-scattered guided waves, the two conventions lead to different signs of the coupling coefficient (see Section 5). Of course this only amounts to a different description of the same physical phenomenon. We close this discussion by noting that our choice in Eqs. (2.4) has the important advantage that it ensures that the field components perpendicular to the waveguide plane are independent of the propagation direction. We return to this issue in Section 3.


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Fig. 1. Schematic of the ideal waveguide. The media are interchangeably denoted either by their refractive index (as in this figure) or by their relative dielectric constant.
Since we treat forward- and backward-traveling modes separately in our discussion, the range of the wave number \( \beta \) is restricted to \( \text{Im}(\beta) \geq 0 \), and if \( \beta \) is real, \( \text{Re}(\beta) > 0 \). The waveguide modes are solutions of Maxwell’s equations [Eqs. (2.2)], with \( \mathbf{P} = 0 \), subject to the appropriate boundary conditions, and their functional forms can be found in any discussion of the theory of optical waveguides, including that of Kogelnik.\(^3\) It is well known that the transverse field components for different modes are mutually orthogonal, which allows one to normalize the modes through the relation

\[
\int_{-\infty}^{\infty} dx [\mathbf{E}(x)]^* \mathbf{E}(x) = 2 \delta \delta', \tag{2.6}
\]

where the star (\( \ast \)) denotes complex conjugation. As it stands, Eq. (2.6) applies only to guided modes and is to be appropriately rewritten if radiation modes with continuously varying indices are involved.\(^2\) Our normalization, together with the definition of the fields in Eq. (2.1), is such that a mode with unit amplitude carries a flux of \( \Phi \), where \( \delta \) is in units of watts/meter. It should be noted that the orthogonality relations as in Eq. (2.6) hold strictly only for lossless waveguides. Losses can be included by making use of a more general orthogonality relation, which can be shown to read exactly like Eq. (2.6), except that the transverse component of the magnetic field itself is used rather than its complex conjugate.\(^2\) In using this orthogonality relation, however, we lose the simple interpretation of \( \delta \) as an energy flow per unit length, which proves convenient when we are considering nonlinear waveguide effects. For this reason, we base our analysis on Eq. (2.6), keeping in mind that it can be generalized as described above.

We are now in a position to evaluate the consequences of a source polarization \( \mathbf{P} \), which we initially choose to be of the form

\[
\mathbf{P}^x(x, z) = p^x(x) \delta(z - z'), \tag{2.7}
\]

where \( z' \) is initially a fixed number. Note that once we have derived the generated field associated with such a source polarization, we can immediately find that for a general, but \( y \)-independent, polarization \( \mathbf{P}(x, z) \) by superposition: Choosing \( p^x(x) = \mathbf{P}(x, z) \), we find that

\[
\mathbf{P}(x, z) = \int_{-\infty}^{\infty} dz' \mathbf{P}^x(x, z), \tag{2.8}
\]

with the fields generated by \( \mathbf{P}(x, z) \) following from the fields generated by the \( \mathbf{P}^x(x, z) \) by a similar superposition. Since initially we keep \( z' \) fixed below, we put \( p^x(x) = p(x) \) for simplicity.

To find the solutions to Maxwell’s equations for a source polarization of the form of Eq. (2.7) we make use of the following trial functions for the fields: \(^\text{12}\)

\[
\mathbf{E}(r) = \delta(z - z') \mathbf{E}_+(x, z) + \delta(z' - z) \mathbf{E}_-(x, z) + \delta(x) \delta(z - z'),
\]

\[
\mathbf{H}(r) = \delta(z - z') \mathbf{H}_+(x, z) + \delta(z' - z) \mathbf{H}_-(x, z), \tag{2.9}
\]

where \( \delta \) is the unit step function, \( \delta(z) = 1 \) for \( z > 0 \), and \( \delta(z) = 0 \) for \( z < 0 \). In Eqs. (2.9) the Dirac delta function is found necessary to balance the \( z \) component of the source polarization [see Eq. (2.10a)]. The generated fields \( \mathbf{E}_\pm \) and \( \mathbf{H}_\pm \) must satisfy Eqs. (2.2) with \( \mathbf{P} = 0 \) and can thus be written as a superposition of normal modes. Since the source polarization is localized at \( z = z' \), \( \mathbf{E}_+ \) (\( \mathbf{E}_- \)) and \( \mathbf{H}_+ \) (\( \mathbf{H}_- \)) must correspond to running waves in the positive (negative) \( z \) direction, or to evanescent waves vanishing as \( |z| \to \infty \); that is, \( \mathbf{E}_+ \) and \( \mathbf{H}_+ \) (\( \mathbf{E}_- \) and \( \mathbf{H}_- \)) must consist of a sum of forward- (backward-) traveling modes, where by sum we mean a sum over the guided modes and an integral over the radiation modes. Substituting Eqs. (2.7) and (2.9) into Eqs. (2.2), we find that the fields must satisfy\(^\text{13}\)

\[
\delta(x) = -p(x)/\epsilon(x) \tag{2.10a}
\]

and

\[
\int_{-\infty}^{\infty} [\mathbf{E}_+(x, z') - \mathbf{E}_-(x, z')] = -\frac{\delta}{dx} \int_{-\infty}^{\infty} \frac{p(x)}{\epsilon(x)}, \tag{2.10b}
\]

Note that on the left-hand sides of Eqs. (2.10b) only the transverse field components contribute. We now project out onto a specific guided mode of the system, say, a TE mode. To do this, we take the scalar product of the first of Eqs. (2.10b) with \( \mathbf{H}^*_m \). We then write \( \mathbf{E}_m \) as a superposition of modes [Eqs. (2.4)] traveling in the appropriate directions, integrate over \( z \), and make use of the orthogonality relation [Eq. (2.6)]. Repeating this procedure for the scalar product of the complex conjugate of the second of Eqs. (2.10b) with \( \mathbf{E}^*_m \), we find two linear equations in the amplitudes of the forward- and backward-traveling modes. Solving for these amplitudes, we then find\(^\text{13}\)

\[
a_n = \frac{i\omega}{\delta} \int_{-\infty}^{\infty} dx \mathbf{E}_m^*(x) \cdot \mathbf{p}(x). \tag{2.11}
\]

We can find similar expressions for the amplitudes \( a_n^\ast \) by projecting Eqs. (2.10b) onto an arbitrary TM-polarized mode. Repeating the procedure described above, and in addition making use of the \( z \) component of Eqs. (2.2b), we find that

\[
a_n = \frac{i\omega}{\delta} \int_{-\infty}^{\infty} dx [\pm \mathbf{e}_m^*(x) \cdot \mathbf{p}(x) + \mathbf{e}_m^\ast(x) \cdot \mathbf{p}(x)]. \tag{2.12}
\]

It is now convenient to combine Eqs. (2.11) and (2.12) by defining

\[
\mathbf{e}_m^\ast(x) = \mathbf{e}_m^*(x),
\]

\[
\mathbf{e}_m^\ast(x) = \mathbf{e}_m^*(x) \pm \mathbf{e}_m^\ast(x) \cdot \mathbf{p}(x). \tag{2.13}
\]

This allows us then to write

\[
a_n = \frac{i\omega}{\delta} \int_{-\infty}^{\infty} dx \mathbf{e}_m^*(x) \cdot \mathbf{p}(x), \tag{2.14}
\]

where the index \( n \) denotes both TE and TM modes.

With Eqs. (2.10a) and (2.14) and the general form of the waveguide modes as in Eqs. (2.4) we have determined the fields due to a source polarization given by Eq. (2.7). As noted above, by superposition we can thus immediately find the fields due to an arbitrary, but \( y \)-independent, source polarization \( \mathbf{P}(x, z) \). Substituting Eqs. (2.4), (2.10a), and (2.14) into the first of Eqs. (2.9), and making use of the superposition principle [Eq. (2.8)], we find that the generat-
ed electric field due to this more general source polarization is given by
\[
\mathbf{E}(x, z) = -\frac{\mathbf{P}(x, z) \cdot \hat{z} \times \hat{z}}{\varepsilon(x)} + \frac{i \omega}{\delta} \sum_n \int_0^\infty d\zeta' \int_0^\infty d\zeta'' \times \left[ \exp[i\beta_n(z - z')] \delta(z - z') \mathbf{e}_n^+(x) \mathbf{e}_n^-(x')^* + \exp[-i\beta_n(z - z')] \delta(z' - z) \times \mathbf{e}_n^-(x) \mathbf{e}_n^+(x')^* \right] \mathbf{P}(x', z'),
\]
where the summation includes all guided modes and should be extended to include an integral over all radiation modes as well. The latter summation is not explicitly written out since radiation modes, and their contribution to the total electric field, are not of direct concern to us in this paper.

Because we eventually apply our method to periodic gratings, we prefer to describe the source polarization in the spatial-frequency domain. With this in mind, we consider a source polarization of the form
\[
\mathbf{P}(x, z) = \mathbf{P}(x, k) \exp(i k \mathbf{z}),
\]
which, when substituted into Eq. (2.15), allows us to evaluate the integral over the dummy variable \( z' \). This then leads to an electric field of the form
\[
\mathbf{E}(x, z) = \mathbf{E}(x, k) \exp(i k \mathbf{z}),
\]
where
\[
\mathbf{E}(x, k) = \int_0^\infty d\zeta' \mathbf{G}(x, x'; k) \cdot \mathbf{P}(x', k),
\]
and the Green function \( \mathbf{G}(x, x'; k) \) is given by
\[
\mathbf{G}(x, x'; k) = -\frac{^{\hat{x}z}(x - x')}{\varepsilon(x)} + i \frac{\omega}{k} \sum_n \left[ \frac{\mathbf{e}_n^+(x) \mathbf{e}_n^+(x')^*}{k_x - \beta_n} - \frac{\mathbf{e}_n^-(x) \mathbf{e}_n^-(x')^*}{k_x + \beta_n} \right].
\]

It is clear from the derivation that this Green function is valid only for (periodic) source polarizations that are independent of the \( y \) coordinate. In Section 3 we lift this restriction and derive the Green function for an arbitrary source polarization.

### 3. GENERAL GREEN FUNCTION

In this section we generalize the result from Section 2. This generalization proceeds in two steps. In the first of these, the Green function in Eq. (2.19) is rewritten in coordinate-free form. This does not involve a coordinate transformation but rather is a way of rewriting the function without an explicit reference to the coordinate frame. So, in a sense, this is just a cosmetic change. It allows us, however, in the second step to argue that the expression thus obtained holds for any direction, as the ideal waveguide is symmetric in the \( y-z \) plane. It is not possible to base this argument directly on Eq. (2.19). Finally, then, the generated fields due to an arbitrary source polarization are obtained by superposition.

To write Eqs. (2.16)–(2.19) in coordinate-free form, we begin by introducing the wave vector \( \mathbf{k} = k \hat{z} \). We denote the magnitude \( |\mathbf{k}| \) by \( k \) and label the unit vector in the direction of \( \mathbf{k} \) by \( \hat{k} = \mathbf{k}/k \). Our first task is to write the bracketed term in Eq. (2.19) in a form that depends explicitly on \( k \) rather than on \( k_x \). Now for \( k_x > 0 \) (\( \hat{k} \) and \( \hat{z} \) point in the same direction) we have \( \hat{k} = \hat{z} \), while for \( k_x < 0 \) (\( \hat{k} \) and \( \hat{z} \) point in opposite directions) we have \( \hat{k} = -\hat{z} \). We now introduce a choice of TE and TM modes different from those in Eqs. (2.13). For TM modes we define
\[
f_{\perp}(x, \hat{k}) = k_e \mathbf{e}_n^+(x) \pm k_h \mathbf{e}_n^-(x),
\]
where \( k_e(x) \) and \( k_h(x) \) are the scalar functions derived in Eqs. (2.25). Comparing Eq. (3.1) with the second of Eqs. (2.13), we see that for \( k_z > 0 \) (\( \hat{k} \) is parallel to \( \hat{z} \); see above) we have \( f_{\perp}(x, \hat{k}) = \mathbf{e}_n^+(x) \), while for \( k_z < 0 \) (\( \hat{k} \) is antiparallel to \( \hat{z} \)) we have \( f_{\perp}(x, \hat{k}) = \mathbf{e}_n^-(x) \). So in either case we have
\[
\frac{k_x}{k} \mathbf{e}_n^+(x) \mathbf{e}_n^+(x')^* - \frac{k_x}{k} \mathbf{e}_n^-(x) \mathbf{e}_n^-(x')^* = \frac{f_{\perp}(x, \hat{k}) f_{\perp}(x', \hat{k})^* - f_{\perp}(x, \hat{k})^* f_{\perp}(x', \hat{k})}{k + k_x}. \tag{3.2}
\]
If \( n \) refers to a TM mode. A similar approach works for TE modes. Putting
\[
f_{\parallel}(x, \hat{k}) = \pm (\hat{k} \times \hat{x}) \mathbf{e}_n^+(x), \tag{3.3}
\]
we find that, if \( k_z > 0 \) (\( \hat{k} \) is perpendicular to \( \hat{z} \)), \( f_{\parallel}(x, \hat{k}) = \pm \mathbf{e}_n^+(x) \) [cf. Eqs. (2.13)], whereas if \( k_z < 0 \) (\( \hat{k} \) is antiperpendicular to \( \hat{z} \)), \( f_{\parallel}(x, \hat{k}) = \mp \mathbf{e}_n^+(x) \), which we may write as \( \mp \mathbf{e}_n^+(x) \). So again we find Eq. (3.2) if \( n \) refers to a TE mode. Putting \( R = y \hat{y} + z \hat{z} \), we may now rewrite Eqs. (2.16)–(2.19): For a one-dimensional source polarization
\[
\mathbf{P}(x, R) = \mathbf{P}(x, k) \exp(ik \cdot R), \tag{3.4}
\]
we have a generated electric field of the form
\[
\mathbf{E}(x, R) = \mathbf{E}(x, k) \exp(ik \cdot R), \tag{3.5}
\]
where
\[
\mathbf{E}(x, k) = \int_0^\infty d\zeta' \mathbf{G}(x, x'; k) \cdot \mathbf{P}(x', k), \tag{3.6}
\]
with the Green function now given by
\[
\mathbf{G}(x, x'; k) = -\frac{k \delta(x - x')}{\varepsilon(x)} + i \frac{\omega}{k} \sum_n \left[ \frac{f_{\parallel}(x, \hat{k}) f_{\perp}(x', \hat{k})^* - f_{\perp}(x, \hat{k}) f_{\parallel}(x', \hat{k})^*}{k + k_x} \right]. \tag{3.7}
\]
and we have made use of the fact that, for either sign of \( k_z \), \( \hat{k} \bot \hat{h} \hat{k} \). Equation (3.7) is equivalent to Eq. (2.19) but has the advantage of being written without explicit reference to a coordinate frame. Note, however, that either term on the left-hand side of Eq. (3.2) can be the more significant as \( |k_x| \rightarrow \beta_n \) near a guided mode, depending on whether the source term drives a forward-propagating mode \( (k_x > 0) \) or a backward-propagating mode \( (k_x < 0) \). In contrast, the more significant term on the right-hand side of Eq. (3.2) is always the first. In fact, in Section 4 we demonstrate that the terms involving the \( f_+ \)’s in Eq. (3.7) play essentially no role at all.

Up to this point we have demonstrated the validity of Eqs. (3.4)–(3.7) only for \( \mathbf{k} = k \hat{z} \); in a sense, we have rewritten the Green function only in terms of the unit vector in the propa-
gation direction rather than in terms of the laboratory coordinate frame. But note that, with the $f_\ell(x, k)$ defined now by Eqs. (3.1) and (3.3), Eqs. (3.5)-(3.7) would still correctly give the generated field if we rotated our original reference frame about the $x$ axis, so that $k$ pointed in an arbitrary direction in the $y$-$z$ plane. Thus those equations hold for an arbitrary $k = k_x \hat{y} + k_z \hat{z}$.

We now generalize these results to determine the generated field due to a general source polarization. In order to do so, we write the source polarization as

$$P(x, R) = \int \frac{dk}{(2\pi)^2} P(x, k)e^{ik \cdot R},$$

(3.8)

where now $k = k_x \hat{y} + k_z \hat{z}$. The generated field is written similarly as

$$E(x, R) = \int \frac{dk}{(2\pi)^2} E(x, k)e^{ik \cdot R},$$

(3.9)

where $E(x, k)$ is given by Eqs. (3.1), (3.3), (3.6), and (3.7). Since an arbitrary source polarization can be represented by the Fourier expansion [Eq. (3.8)], we have derived an expression for the field generated by an arbitrary source polarization.

We finish this section with a brief comment on Eqs. (3.1) and (3.3). In these equations we have defined the relevant waveguides modes [the $f_\ell$; see the paragraph following Eq. (3.7)] in a unified way in the sense that they hold for any propagation direction. Physically this expresses the notion that with the introduction of modes traveling in arbitrary directions in the waveguide plane, the strict distinction between forward- and backward-traveling waves as in Eqs. (2.4) is no longer useful. The definitions in Eqs. (3.1) and (3.3) imply that the directions of the electric fields are uniquely determined by the propagation direction. For counterpropagating modes of unit amplitude, for example, Eqs. (3.1) and (3.3) lead to oppositely directed field components in the waveguide plane.

4. PRINCIPAL POLE APPROXIMATION

As mentioned, the Green function derived in Section 3 gives the response of the waveguide to an arbitrary source polarization. It is important to recall at this point that the Green function is frequency dependent not only through the factor $\omega$ in the second term in Eq. (3.7) but also through the frequency dependence of the dielectric function in the first term and that of the normal modes in the second. Although we have suppressed this dependence in previous sections to avoid cluttered notation, it is henceforth necessary to write it explicitly.

The Green function [Eq. (3.7)] contains a summation over all normal modes, including both guided and radiation modes [see the discussion following Eqs. (2.4)]. However, in many practical situations only a few modes are significantly excited. In terms of our discussion in Section 1, this means that only a few modes are significantly influenced by the waveguide imperfections. In this case we do not need all the information provided by the Green function, and we therefore use an approach in which the general Green function is approximated by series expansions in the spatial- and temporal-frequency domains about the points of main interest. Obviously, by including enough of these points of interest and by including more and more terms in the expansions, one can approximate the Green function to an arbitrary degree of accuracy. In practice, a sensible balance between the desire for high accuracy and that for computational convenience has to be found.

As a first step in our approximation, we consider the part of the source polarization that is peaked near $k_0$ and $\omega_0$, where

$$h_0 = \beta_n(\omega_0),$$

(4.1)

for some particular mode $n$. This assumes that we consider guided modes only. The theory can be extended to include radiation modes as well, but we will not do so here. Restricting ourselves to the electric field generated by the part of the polarization peaked about such a $(k_0, \omega_0)$, we first simply multiply Eq. (3.7) by $[k - \beta_n(\omega)]$ to obtain

$$[k - \beta_n(\omega)]E(x, k; \omega) = \int \frac{dk}{2\pi} P(x', k; t) \exp(ik \cdot R),$$

(4.2)

Next we make use of the fact that $P(x', k; \omega)$ is nonnegligible only for $k$ and $\beta_n(\omega)$ close to $k_0$ and $\beta_n(\omega_0)$, respectively, to simplify Eq. (4.2) in two ways. First, because of Eq. (4.1), $k - \beta_n(\omega)$ is small, and we conclude that the only sizable contribution from the right-hand side of Eq. (4.2) comes from the second term when $n' = n$, so that we can drop the contributions of all other modes. Second, we neglect the dependence of the only remaining waveguide mode on $k$ and $\omega$ and thus freeze it at $k_0$ and $\omega_0$. These two approximations then allow us to simplify Eq. (4.2) to

$$[k - \beta_n(\omega)]E(x, k; \omega) = \int \frac{d\omega}{2\pi} P(x', k; \omega_0) \exp(-i\omega t),$$

(4.3)

Defining now the temporal Fourier transform of the fields as

$$P(x, k; t) = \int \frac{d\omega}{2\pi} P(x, k; \omega) \exp(-i\omega t),$$

(4.4a)

$$E(x, k; t) = \int \frac{d\omega}{2\pi} E(x, k; \omega) \exp(-i\omega t),$$

(4.4b)

and using Eqs. (3.8) and (3.9), we can rewrite Eq. (4.3) as

$$\int \frac{dk}{(2\pi)^2} \int \frac{d\omega}{2\pi} [k - \beta_n(\omega)]E(x, k; \omega) \exp(ik \cdot R) \exp(-i\omega t)$$

$$= \int \frac{d\omega}{2\pi} \int \frac{dk}{(2\pi)^2} \frac{e^{ik \cdot (x - R)}}{2\pi} \frac{\omega_0}{\omega} f'_n(x, k_0; \omega_0) \cdot f''_n(x', k'_0; \omega_0) \cdot P(x', R; t).$$

(4.5)

Next we introduce a slowly varying source polarization $P$ by taking out the rapidly varying components in the following way:
where the subscript refers to k_0 and ω_0. This allows us to rewrite Eq. (4.5) as

\[ -i \int \frac{dk}{(2\pi)^2} \int \frac{dω}{2\pi} [k - β_n(ω)] E(x, k; ω) \exp[i(k - k_0) \cdot R] \times \exp[-i(ω - ω_0)t] \]

\[ = \frac{ω_0}{δ} \int_{-∞}^{∞} dx' f_4(x', k_0; ω_0) \times f_1(x', k_0; ω_0)^* \cdot P_0(x', R; t). \quad (4.7) \]

We can now proceed to our final approximations, which involve the left-hand side of Eq. (4.7). By Eq. (4.1), the term in brackets in the integrand of Eq. (4.7) can be written as

\[ (k - k_0) - [β_n(ω) - β_n(ω_0)], \quad (4.8) \]

where we approximated the inverse of the group velocity of mode n at frequency ω_0. From now on we denote the dimensionless quantity in brackets in expression (4.9) by δ_n. Obviously, the approximation leading from expression (4.8) to expression (4.9) can be improved by including more terms in the expansion—the next term would involve the group-velocity dispersion of mode n at ω_0. The accuracy of expression (4.9), however, is sufficient for the present purposes. It is now crucial to recognize that in the integrand on the left-hand side of Eq. (4.7) we have the equivalence

\[ \frac{∂}{∂t} \Leftrightarrow -i(ω - ω_0), \quad (4.10) \]

so that we can make the following replacement:

\[ \frac{η_0 n_0}{c} (ω - ω_0) \Leftrightarrow - \frac{iη_0 n_0}{c} \frac{∂}{∂t}, \quad (4.11) \]

and the time derivative may be taken out of the integral in Eq. (4.7). In doing this, we implicitly assume that E(x, k; ω) vanishes fast enough as ω \to ±∞. In practice, this requirement is always satisfied. Note that for strictly monochromatic source polarizations at ω_0 we produce strictly monochromatic fields at ω = ω_0, so that the operator in formula 4.11 can be made to vanish.

We can treat the term k - k_0 in expression 4.9 in a similar way, with the complication, however, that the k is two dimensional. Since k = [(k_0 - δ_0)^2 + |k_0|^2]^{1/2}, and k is close to k_0, we can show by a Taylor series expansion that to lowest order

\[ (k - k_0) \approx -i \left( k_0 \cdot \nabla - i \frac{|k_0|^2}{2k_0} \right), \quad (4.12) \]

where

\[ \nabla = \frac{∂}{∂y} + 2 \frac{∂}{∂z} \quad (4.13) \]

operates in the plane of the waveguide only. By an argument similar to that above, the expression on the right-hand side of formula (4.12) may be taken out of the integral in Eq. (4.7) as well. It should be noted (see below) that the first term on the right-hand side of formula (4.12) describes the forward propagation of the waveguide mode, whereas the second term describes finite beam effects. The latter term can be dropped if one is interested only in the propagation of waveguide modes that can be assumed to have uniform amplitude in the direction in the y-z plane perpendicular to that of propagation. It can be shown that the approximations leading to formula (4.12) are equivalent to the well-known Fresnel approximation in diffraction problems.

Substituting relations (4.8), (4.11), and (4.12) into Eq. (4.7), we then find that approximately

\[ -i Ω \int \frac{dk}{(2π)^2} \int \frac{dω}{2π} E(x, k, ω) \exp[i(k - k_0) \cdot R] \times \exp[-i(ω - ω_0)t] \]

\[ = \frac{ω_0}{δ} \int_{-∞}^{∞} dx' f_4(x', k_0; ω_0) \times f_1(x', k_0; ω_0)^* \cdot P_0(x', R; t), \quad (4.14) \]

where the operator

\[ Ω \equiv k_0 \cdot \nabla - i \frac{|k_0|^2}{2k_0} + \frac{η_0 n_0}{c} \frac{∂}{∂t}. \quad (4.15) \]

We next introduce the slowly varying function ε_n through the relation [cf. Eq. (4.6)]

\[ E(x, R; t) = ε_n(k_0, R; t) f_1(x', k_0; ω_0) \times \exp(ik_0 \cdot R) \exp(-iω_0 t) + c.c., \quad (4.16) \]

so that we can finally rewrite Eq. (4.14) in terms of slowly varying functions:

\[ Ω ε_n(k_0, R; t) = i \frac{ω_0}{δ} \int_{-∞}^{∞} dx' f_4(x', k_0; ω_0)^* \cdot P(x', R; t). \quad (4.17) \]

Equation (4.17) holds for an arbitrary source polarization peaked about k_0 and ω_0 satisfying Eq. (4.1) and is the final result of this section: It shows how the amplitude of the waveguide mode at (k_0, ω_0) is modified by this polarization. In Section 5 we apply this general result to describe mode coupling in perturbed waveguides.

5. COUPLED-MODE DESCRIPTION AND CONCLUSIONS

We are now in a position to consider our main problem of interest, the mutual coupling of ideal waveguide modes owing to deviations in the guide from the ideal. To illustrate how the argument proceeds, we assume that the film-cladding interface is periodically modulated as shown in Fig. 2. Note that the perturbed structure is chosen such that an additional dielectric constant (ε_b) is introduced into the geometry. The situation in Fig. 2 describes the grating profiles recently described by Reider et al.12 For more conventional waveguides the profile is sinusoidal, and ε_b = ε_f. We refer to the region of space where the dielectric constant differs from that in the ideal waveguide structure as the perturbed region, and we assume that the thickness δh of
The waveguide structure mentioned above allows us to make two waveguide perturbations in the presence of the field in Eq. (5.1). In the absence of such perturbations, the waveguide structure is perpendicular to the $z$ axis. Material is much less than suggested. The grating rulings are perpendicular to the $z$ axis.

Fig. 2. Schematic of the grating structure that we consider. The figure is slightly misleading since the thickness of the deposited material is much less than suggested. The grating rulings are perpendicular to the $z$ axis.

This perturbed region is much smaller than both the film thickness and the wavelength of light. Since $d$ is typically of the order of the wavelength of light, this implies that $\delta h \ll d$.

Since the waveguide modes form a complete, orthogonal set, we can always write the field outside the perturbed region of the waveguide as

$$E(x, R; t) = \sum_{q} \delta_{q}(k_{j}, R; t) f_{q}^{+}(x, \kappa_{j} \omega_{j})$$

$$\times \exp(ik_{j} \cdot R) \exp(-i\omega_{j}t) + c.c.,$$  \hspace{1cm} (5.1)

where the sum over waveguide modes $q$ with directions of propagation $k_{j}$ includes both the modes originally excited and those then generated by the deviations from an ideal waveguide structure. In the absence of such perturbations, of course, the $\delta_{q}$ would be independent of $R$ and $t$. By calculating the source polarization that results from the waveguide perturbation in the presence of the field in Eq. (5.1), we can then use Eq. (4.17) to find coupled equations for the different mode amplitudes $\delta_{q}$.

The assumption of a small perturbation from the ideal waveguide structure mentioned above allows us to make two approximations. The first of these is that, to first order, the change in the fields from the ideal to the perturbed guide vanishes everywhere except in the perturbed region. This implies, for example, that the electric field in the waveguide in Fig. 2 is, to first order, different from that in Eq. (5.1) only where $\epsilon = \epsilon_{0} \epsilon_{0}$. Second, since $\delta h$ is much less than the wavelength of light, the actual electric field varies essentially not at all with $x$ in the perturbed region. In the example of Fig. 2 it can thus be found from the field in the ideal waveguide [Eq. (5.1)] by applying the Maxwell saltus conditions across the interface where the relative dielectric constant changes from $\epsilon_{0}$ to $\epsilon_{0}$ (or where $\epsilon_{0}$ changes to $\epsilon_{0}$).

From an initial assumption of a field as in Eq. (5.1), the two approximations outlined above allow us to find the electric field that, to first order, is present in the perturbed region and hence the polarization there. We can find the appropriate source polarization by then determining the value that, in the ideal guide, would give the same change in polarization as that associated with the perturbation in the actual guide. Since the Green function gives the response of the ideal waveguide to any source polarization, we have then solved our problem. In the example of Fig. 2 this leads to source polarization:

$$\mathbf{P}(x, R; t) = \sum_{q} \tilde{s}(x, R) \cdot \delta_{q}(k_{j}, R; t) f_{q}^{+}(x, \kappa_{j} \omega_{j})$$

$$\times \exp(ik_{j} \cdot R) \exp(-i\omega_{j}t) + c.c.,$$  \hspace{1cm} (5.2)

where the perturbation tensor $\tilde{s}(x, R)$ is given by

$$\tilde{s}(x, R) = \left\{ \begin{array}{ll} \epsilon_{0}(\epsilon_{0} - \epsilon_{0}) \\ \epsilon_{0} \kappa_{j} \gamma + \epsilon_{0} \kappa_{j} \gamma + \epsilon_{0} \kappa_{j} \gamma) \\ 0 & \text{if } \epsilon = \epsilon_{0} \epsilon_{0} . \\ & \text{otherwise} \end{array} \right.$$  \hspace{1cm} (5.3)

The detailed determination of the perturbation tensor from the arguments given above is presented in Appendix A. It should be stressed that our approach is invalid when the grating in Fig. 2 is too deep. The analysis then has to be refined, for example by making use of an iterative procedure, but we will not do so here.

We can now find the coupled-mode equations by substituting Eq. (5.2) into Eq. (4.17), which results in

$$\mathcal{O} \delta_{q}(k_{0}, R; t) = i \sum_{q} C_{q}^{q}(R) \exp[-i(k_{0} - k_{q}) \cdot R]$$

$$\times \exp[i(\omega_{0} - \omega_{j})t] \delta_{q}(k_{j}, R; t),$$  \hspace{1cm} (5.4)

where $\mathcal{O}$ was defined in Eq. (4.15) and the coupling term is defined to be

$$C_{q}^{q}(R) = \frac{\omega_{0}}{\delta} \int_{-\infty}^{\infty} dx f_{q}^{+}^{*}(x, \kappa_{q} \omega_{q}) \cdot \bar{s}(x, R) \cdot f_{q}^{+}(x, \kappa_{j} \omega_{j}).$$  \hspace{1cm} (5.5)

Allowing for a different normalization of the waveguide modes, Eqs. (5.4), (5.5), and (4.15) reduce to the well-known coupled-mode equations in the limit in which $\omega_{0} = \omega_{j}$. We now evaluate Eq. (5.5) to find the coupling between two waveguide modes through a diffraction grating. We do not consider coupling involving TE modes in detail, as all methods of waveguide analysis (see Section 1) agree on such coupling processes. Instead, we just give the final results later in this section, turning now to consider the coupling between two TM modes. As the procedure to evaluate expressions such as Eq. (5.5) is well understood, we leave our analysis here brief. The evaluation requires the functional form of the TM$_{0}$ mode of the ideal waveguide as in Fig. 1, for which we refer, e.g., to Ref. 3, whose notation we have adopted here. The only difference is our normalization of the waveguide modes [Eq. (2.6)], which leads to a slightly different expression for the mode amplitudes.

To start our evaluation of Eq. (5.5), we consider a forward-propagating TM$_{0}$ mode in the $z$ direction, normally incident onto a grating structure as in Fig. 2, and consider the coupling with a backward-scattered TM$_{0}$ mode. For this case, from Eq. (3.1),

$$f_{q}^{+}(x, \kappa_{q} \omega_{q}) = \hat{f}_{q}^{\pi}(x) + \hat{f}_{q}^{\pi}(x),$$

$$f_{q}^{+}(x, \kappa_{j} \omega_{j}) = \hat{f}_{q}^{\pi}(x) - \hat{f}_{q}^{\pi}(x).$$  \hspace{1cm} (5.6)

As was argued above, the waveguide is assumed to be perturbed only in the region where the extra material has
been deposited. We describe the deposition by the function $\text{rect}(z/d)$, which is defined as

$$\text{rect}(z/d) = \begin{cases} 1 & (4n - 1)/4 \leq z/d < (4n + 1)/4 \\ 0 & (4n + 1)/4 \leq z/d < (4n + 3)/4 \end{cases}$$

$$n = 0, \pm 1, \pm 2, \ldots \quad (5.7)$$

Using Eqs. (5.3) and (5.5)-(5.7), and dropping the subscripts and superscripts of $C$, we then find that

$$C(z) = \epsilon_c (\epsilon_e - \epsilon_c) \text{rect}(\frac{z}{d}) \left[ \frac{\omega_0}{\delta} \right]^{N_{TM}} \frac{e^{i \phi h + i \phi h}}{h_{\text{eff,TM}}} \frac{1}{n_c n_e} \frac{n_e^2 - n_c^2}{n_{TM}^2} \frac{1}{q_c} \frac{n_b^2 - n_c^2}{q_e^2} \text{rect}(z/d),$$

where the $x$ dependence of the modes was dropped and $h$ is the thickness of the guiding film. Because of the directions of the electric field associated with TM modes, we see that the $\delta \theta$ component of $\sigma(x, R)$ [Eq. (5.8)] does not contribute. The $\delta \phi$ component contributes

$$\frac{2\pi}{\lambda} \frac{n_b^2 - N_{TM}^2}{N_{TM}} \frac{\delta \theta}{h_{\text{eff,TM}}} \left[ 1 - \frac{N_{TM}^2}{n_c^2} \right] \frac{1}{q_c} \frac{n_b^2 - n_c^2}{q_e^2} \frac{n_b^2 - n_c^2}{n_e^2} \frac{1}{q_e} \frac{n_b^2 - n_c^2}{q_e} \frac{n_b^2 - n_c^2}{n_e^2} \text{rect}(z/d),$$

whereas the $\delta \phi$ component yields

$$\frac{2\pi}{\lambda} \frac{n_b^2 - N_{TM}^2}{N_{TM}} \frac{\delta \theta}{h_{\text{eff,TM}}} \left[ 1 - \frac{N_{TM}^2}{n_c^2} \right] \frac{1}{q_c} \frac{n_b^2 - n_c^2}{q_e^2} \frac{n_b^2 - n_c^2}{n_e^2} \frac{1}{q_e} \frac{n_b^2 - n_c^2}{q_e} \frac{n_b^2 - n_c^2}{n_e^2} \text{rect}(z/d),$$

where $q_i$ were replaced by $n_i^2$. Further, $\delta \phi$ is defined in Fig. 2. $N_{TM}$ is the guide index of the TM0 mode, $h_{\text{eff,TM}}$ is the effective waveguide width, and $q_i$ is an auxiliary function of the various refractive indices and of the guide index, which is given by

$$q_i = \frac{N_{TM}^2}{n_c^2} + \frac{N_{TM}^2}{n_e^2} - 1.$$  

$$\quad (5.11)$$

The total TM$\alpha$-TM$\beta$ coupling term for normal incidence is the sum of the two expressions. We now immediately consider the case of nonnormal angles of incidence (see Fig. 3). The $x$ components of the electric field do not depend on the direction of propagation, so that expression (5.10) carries over directly. From Fig. 3 we see that the contribution of the in-plane component is modified by a factor $\cos(2\theta)$. This factor describes the well-known increases in the effective interaction region for grazing angles of incidence. In Ref. 7 this effect is included in the coupling coefficients themselves, whereas here it is implicitly contained in the coupled-mode equations. To see where it appears in our formalism we turn to the operator $\sigma$ [Eq. (4.15)], which, when written out explicitly, gives the required factor multiplying the $\delta \partial z$ term.

In substituting this result into the coupled-mode equations one usually uses only the Fourier component of $C$ that (almost) Bragg matches the two modes. We note in passing that Eq. (5.12) is valid for both positive and negative values of $\delta \theta$.

We now consider the more common waveguide grating in which the perturbation is sinusoidal and $n_b = n_f$ and make the appropriate substitutions in Eq. (5.12) [since the rect function from Eq. (5.7) is even, it should be replaced by a cosine]. To compare the results with existing literature, however, we need the coupling coefficient $\kappa$, which is defined through the relation

$$C = \kappa \left[ \exp(2\pi i z/d) + \exp(-2\pi i z/d) \right]$$

and is thus the factor preceding the relevant (i.e., Bragg matched) Fourier component. With the substituents mentioned above, we find immediately from Eqs. (5.12) and (5.13) that

$$\kappa_{\text{TM-TM}} = \frac{\pi}{\lambda} \frac{n_f^2 - N_{TM}^2}{N_{TM}} \frac{\Delta h}{h_{\text{eff,TM}}} \left[ 1 - \frac{N_{TM}^2}{n_c^2} \cos(2\theta) \right] \frac{1}{q_c} \frac{n_b^2 - n_c^2}{q_e^2} \frac{n_b^2 - n_c^2}{n_e^2} \text{rect}(z/d).$$

$$\quad (5.14)$$

where $\Delta h$ denotes the amplitude of the sinusoidal perturbation. Equation (5.14) is in agreement with the results of a total field analysis, and, apart from a factor $1/\cos(\theta)$, with that of the local normal mode expansion \cite{7} as well. This factor describes the well-known increases in the effective interaction region for grazing angles of incidence. In Ref. 7 this effect is included in the coupling coefficients themselves, whereas here it is implicitly contained in the coupled-mode equations. To see where it appears in our formalism we turn to the operator $\sigma$ [Eq. (4.15)], which, when written out explicitly, gives the required factor multiplying the $\delta \partial z$ term.

For the sake of completeness we now finally also give results for the coupling between two $\text{TE}_0$ modes,

$$\kappa_{\text{TE-TE}} = \pi \frac{n_f^2 - N_{TE}^2}{\lambda} \frac{\Delta h}{h_{\text{eff,TE}}} \cos(2\theta),$$

$$\quad (5.15)$$

\[
\begin{array}{c}
\end{array}
\]
and those for the coupling between $TM_0$ and $TE_0$:

$$\kappa_{TE-TM} = \kappa_{TM-TE} = \frac{\pi}{\lambda} \frac{\Delta n}{(n_{TM}^2 - n_0^2)^{1/2}} \frac{1}{q_c} \left( \frac{n_i}{n_c} \right)^{1/2} \frac{(N_{TM}^2 - n_0^2)^{1/2}}{N_{TM}} \sin(\theta_{TM} + \theta_{TE}), \quad (5.16)$$

which follow from our formalism as well. Note that the sign of $\kappa_{TE-TM}$ is opposite that of the commonly quoted expression.\textsuperscript{2,3,5,7} As was discussed in the paragraph following Eqs. (2.5), this is due to a different convention of the relative signs of the fields of forward- and backward-traveling TE modes, which must be compensated for by an opposite amplitude of the backward-traveling modes to describe the same fields. Of course, this influences not the physics but rather the way in which we describe it.

In conclusion, we have presented a new procedure to describe mode coupling in perturbed waveguides, which is based on the modes of the ideal waveguide. We find an expression for the TM-TE coupling coefficient for a waveguide diffraction grating that is in agreement with experiments,\textsuperscript{11} in contrast to ideal mode expansions, which are commonly used.\textsuperscript{1,3,5,6} Our method also allows us to analyze conveniently waveguide geometries into which an extra index of refraction is introduced. In addition, our method quite naturally includes finite beam effects, and it allows for a slow time dependence of the fields. This has proved to be quite convenient for studying pulse propagation in nonlinear waveguides. We plan to return to this matter in a future publication.

**APPENDIX A**

In this appendix we demonstrate that the perturbation tensor $\tilde{\sigma}(x, R)$ is given by Eq. (5.5). It is straightforward to do this if the waveguide is not covered by a dielectric material. Since a repeated application of the Maxwell saltus conditions at the interfaces then leads to the desired result, we consider this simple case first. After this we treat the more complicated situation in which $\epsilon_2 \neq 1$. To prove Eq. (5.3) for this case, we need to consider the properties of a dipole sheet embedded in a dielectric. Once these have been established in the proper limits, Eq. (5.3) follows by an argument that is similar to that for the simple case $\epsilon_2 = 1$. We henceforth use the nomenclature "new" to denote the fields that are present in the perturbed waveguide, while by "old" we mean the corresponding fields that would be present in the ideal guide.

We first consider waveguides without a polarizable cover. Because of the different saltus conditions for the field components parallel and perpendicular to an interface, we have to treat these components separately. With this in mind we write the polarization in the perturbed region as

$$P_{new} = \epsilon_0(\epsilon_0 - 1)E_{new}^0(\hat{\xi}+\hat{\eta}+\hat{\zeta}), \quad (A1)$$

where $E_{new}^0$ is the field just above the film in the perturbed region. By the second approximation discussed after Eq. (5.1), both $P_{new}$ and $E_{new}^0$ may be taken as constant as $x$ varies in the perturbed region for fixed $y$ and $z$. Now, by the continuity of the transverse component of the electric field and by the normal component of the electric displacement, we have

$$\left(\epsilon_0\hat{\xi}+\hat{\eta}+\hat{\zeta}\right)E_{new}^0 = \left(\epsilon_0\hat{\xi}+\hat{\eta}+\hat{\zeta}\right)E_{old}^0, \quad (A2)$$

where $E_{new}^0$ refers to the field in the film just below the perturbed region, and by the first approximation after Eq. (5.1) we have

$$E_{new}^0 \approx E_{old}^0. \quad (A3)$$

Using the same saltus condition that gave Eq. (A2), we find for the ideal (coverless) waveguide structure that

$$E_{old}^0 = \left(\epsilon_0\hat{\xi}+\hat{\eta}+\hat{\zeta}\right)E_{old}^0, \quad (A4)$$

where $E_{old}^0$ refers to the field in the cladding just above the film in the ideal waveguide. Combining formulas (A1)--(A4), we thus find that

$$P_{new} = \epsilon_0(\epsilon_0 - 1)\left(\frac{1}{\epsilon_0}\hat{\xi}+\hat{\eta}+\hat{\zeta}\right)E_{old}^0, \quad (A5)$$

Since the ideal waveguide is not covered by a dielectric, $P_{old} = 0$. Relation (A5) thus gives the change in the polarization due to the waveguide perturbation, and this must be produced by introducing a source polarization in the cover region of the ideal waveguide structure. We thus find that

$$P_s = \epsilon_0(\epsilon_0 - 1)\left(\frac{1}{\epsilon_0}\hat{\xi}+\hat{\eta}+\hat{\zeta}\right)E_{old}^0, \quad (A6)$$

which leads immediately to the perturbation tensor given in Eq. (5.3) in the limit in which $\epsilon_2 = 1$.

As was mentioned above, the situation in which $\epsilon_2 \neq 1$ requires a more subtle argument, which has to take the dielectric screening of the cover into consideration. For this purpose we first establish a key result that we will need. We consider a thin polarization sheet with thickness $d$ embedded in a uniform medium with relative dielectric constant $\epsilon_2$. Again considering harmonically varying fields with angular frequency $\omega$, we define the field amplitudes according to Eq. (2.1) and write the wave number of the radiation at that frequency as $k = \sqrt{\epsilon_2}/\omega/c$. We take the sheet thin enough that $rd \ll 1$.

The properties of such polarization sheets (not necessarily thin) have been studied extensively in the context of surface optics,\textsuperscript{14--16} and the results in that area can be used in understanding the present problem. Specifically, we use a result previously derived by one of us,\textsuperscript{16} which shows that, for a general source polarization as in Eq. (3.8) in a uniform medium with $\epsilon_2$, the generated field can be written as Eq. (3.9). These are related through a relation as Eq. (3.6), with a Green function that in this simple geometry is given by\textsuperscript{16}

$$\tilde{G}(x, k) = \frac{1}{\epsilon_0} \frac{\omega^2}{2\epsilon_0 c^2} \left(\frac{\delta^3 + \delta_+ \delta_-}{\delta(x)} x e^{i\omega x} \right) \delta(x), \quad (A7)$$

where $w = (\omega^2 - k^2)^{1/2}$ is the $x$ component of the wave vector. The unit vectors $\delta_+$ and $\delta_-$ are perpendicular to the wave vector, but their precise definition does not matter here.
Since $\nu d < 1$ we may drop the exponential factors in Eq. (A7). On integrating this expression over the sheet thickness, we find that the last term within brackets vanishes outside the sheet but is of order unity inside, while the other two terms are everywhere of order $\nu d$, which, by assumption, is much smaller than unity. Under our conditions only the third term in Eq. (A7) is thus relevant. Hence we conclude that all fields essentially vanish outside the sheet, even if the source polarization is oscillating. Inside the sheet we have $\epsilon_0 \epsilon E = -P_s \partial \hat{x}$ according to Eqs. (3.6), (3.8), (3.9), and (A7), so that we have an induced polarization there of

$$P\text{ind} = \epsilon_0 (\epsilon - 1) E = -\epsilon - 1 \frac{\epsilon}{\epsilon} P_s \partial \hat{x}.$$  \hspace{1cm} (A8)

The total polarization that results from the source polarization is given by $P_s + P\text{ind}$, which we denote by $\Delta P$ for reasons that will become clear below. Combining with Eq. (A8), we find

$$\Delta P = P_s \left( \frac{1}{\epsilon_0} \frac{\epsilon}{\epsilon} \hat{x} + \hat{y} + \hat{z} \right).$$ \hspace{1cm} (A9)

We now can turn to the problem at hand. Initially the argument is identical to that in the simple case with $\epsilon_0 = 1$, except that Eq. (A4) now reads as

$$\epsilon_0 \epsilon \hat{x} + \hat{y} + \hat{z} E^{\text{old}} = \epsilon_0 \epsilon \hat{x} + \hat{y} + \hat{z} E^{\text{old}},$$ \hspace{1cm} (A10)

so that instead of relation (A5) we find

$$P^{\text{new}} \approx \epsilon_0 (\epsilon_0 - 1) \left( \frac{\epsilon}{\epsilon} \frac{\epsilon}{\epsilon} \hat{x} + \hat{y} + \hat{z} \right) E^{\text{old}}.$$ \hspace{1cm} (A11)

Now, however, we have

$$P^{\text{old}} = \epsilon_0 (\epsilon_0 - 1) E^{\text{old}},$$ \hspace{1cm} (A12)

so that in the perturbed region

$$\Delta P = P^{\text{new}} - P^{\text{old}} \approx \epsilon_0 (\epsilon_0 - \epsilon_0) \left( \frac{\epsilon}{\epsilon} \frac{\epsilon}{\epsilon} \hat{x} + \hat{y} + \hat{z} \right) E^{\text{new}}.$$ \hspace{1cm} (A13)

This is the change in the polarization due to the waveguide perturbation, which we must produce by introducing a source polarization in the cladding region of the ideal waveguide structure. Since $\hbar k d$, the geometry leading to Eq. (A9) is appropriate. Combining Eq. (A9) and relation (A13), we find that we should set the source polarization equal to

$$P_s = \epsilon_0 (\epsilon_0 - \epsilon_0) \left( \frac{\epsilon}{\epsilon} \frac{\epsilon}{\epsilon} \hat{x} + \hat{y} + \hat{z} \right) E^{\text{old}}.$$ \hspace{1cm} (A14)

This expression agrees with relation (A6) when $\epsilon_0 = 1$ and immediately leads to Eq. (5.3) for the perturbation tensor in the general case.

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**REFERENCES AND NOTES**


17. This conclusion is not valid if $w$ is too small or if $|\lambda| \approx \nu$. However, a polarization with period $2\pi/\nu$ can never couple two counterpropagating guided modes.