Self-localized light: launching of low-velocity solitons in corrugated nonlinear waveguides

C. Martijn de Sterke and J. E. Sipe

Department of Physics, University of Toronto, Toronto, Ontario M5S 1A7, Canada

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We show that stationary gap solitons are just a limiting case of a wider set of (slowly) moving solitons associated with the photonic stop gap of a periodic nonlinear structure. We give an analytic description of these solitons and provide the prescription for launching such solitons in a realistic waveguide geometry.

Gap solitons were originally discovered in computer experiments of the harmonic response of nonlinear periodic media at normal incidence. Although the latter restriction was dropped in subsequent numerical investigations, the limitation to harmonic time dependence has consistently been maintained. In a theoretical analysis of this problem, however, we have shown that solitons can occur in nonlinear periodic media under far less restrictive conditions. In fact, the demonstration that the envelope function of the electric field inside the nonlinear periodic structure satisfies the nonlinear Schrödinger equation (NLSE) implies the existence of N-soliton solutions, in which each of these solitons has its own height and velocity. The gap soliton corresponds to the 1-soliton solution of the NLSE, in which this soliton is stationary. In this Letter we make what is to our knowledge the first attempt to study some of the properties of the more general, time-dependent solutions. In doing so, we restrict ourselves to 1-soliton solutions (but with non-zero velocity). By choosing the parameters appropriately it is, in principle, possible to obtain solitons with an arbitrary velocity lying between zero and c/f, where c is the average refractive index of the periodic medium.

The solitonic effects in nonlinear periodic structures, however, are purely classical. As is well known, solitons arise in general from a delicate balance between dispersive and nonlinear effects. Since the low curvature of the photonic band structure at an arbitrary position in the Brillouin zone would give rise to wide solitons, we confine ourselves to the frequency region near one of the stop gaps. We thus consider radiation centered near a frequency that is much smaller than the first term but is essential to describe, for example, energy transport through the structure. It should be mentioned that the additional assumption that the frequency spectrum of the field significantly overlaps only one side of the photonic stop gap; we label the state associated with the band edge at that side m and its Bloch function \( \varphi_m \). The advantage of this additional assumption becomes clear below.

Our mathematical analysis makes use of an envelope-function approach, in which the electric field in the periodic medium, to the lowest order, is approximated as the product of the Bloch function \( \varphi_m(z) \) that varies harmonically at its eigenfrequency \( \omega_m \) and a slowly varying envelope function \( a(z, t) \),

\[
E(z, t) = a(z, t)\varphi_m(z) \exp(-i\omega_m t) + \sum_l \Lambda_{l,m} \frac{\partial a}{\partial z} \varphi_l(z) \exp(-i\omega_{l,m} t) + \text{c.c.,} \tag{1}
\]

where \( d \) is the period of the stack, c.c. denotes complex conjugation, and \( \Lambda_{l,m} \) is a dimensionless coupling coefficient that vanishes unless \( \varphi_m \) and \( \varphi_l \) have the same crystal momentum. The second term in this equation is much smaller than the first term but is essential to describe, for example, energy transport through the structure. It should be mentioned that the additional assumption in the previous paragraph ensures the particularly simple form of the first term in Eq. (1). We have previously shown that the envelope function \( a(z, t) \) satisfies the NLSE,

\[
i \frac{\partial a}{\partial t} + \frac{1}{2} \omega_m'' \frac{\partial^2 a}{\partial z^2} + \alpha_m |a|^2 a = 0, \tag{2}
\]

where \( \omega_m'' \) is the curvature of the photonic band struc-
nature at the edge of the stop gap and $\alpha_m$ is an effective nonlinear coefficient that depends on the overlap of the nonlinearity and the Bloch function $\varphi_m$. In previous analyses, $\alpha_m$ was assumed harmonic time dependence for $\alpha(z, t)$ at this point, which turns the NLSE into an ordinary differential equation. Now, however, we consider the general 1-soliton solution to the NLSE. It should be noted that no such solutions exist unless $\omega_m$ and $\alpha_m$ have the same sign. Under this condition we can write these solutions as:

$$
\alpha(z, t) = A \exp(iBz) \exp[-i(\delta + \Delta)t] \text{sech}(Bz - Bt),
$$

(3)

where

$$
A = \sqrt{-2B/\alpha_m},
$$

$$
B_1 = \sqrt{-2B/\omega_m},
$$

$$
B_2 = \sqrt{2B/\omega_m},
$$

$$
B = \omega_m B_1 B_2,
$$

and the signs of the detunings $\delta$ and $\Delta$ are chosen such that these coefficients come out to be real. Although they are completely arbitrary apart from this restriction, it should be kept in mind that our envelope-function approach requires these detunings to be much smaller than the size of the stop gap. From Eqs. (3) and (4) we see that $\delta$ determines the height and the spatial width of the soliton, whereas $\Delta$ determines the velocity. In the limit in which $\Delta \to 0$, our previous results for harmonic time dependence are recovered.

The geometry in which we study these solitons is as follows: The left half-space, defined by $z < 0$, consists of a uniform linear medium with refractive index $n$. The right half-space, defined by $z > 0$, is occupied by the periodic medium with a refractive index $n(z) = n + (\Delta n) \sin(2\pi z/d)$ and a nonlinear coefficient $n^{(3)}(z)$ with the same period. We now ask the following question: Which pulse, coming from the left in the linear medium, will, on reflection from the interface, launch a soliton with a prescribed height and velocity into the nonlinear stack? To solve this problem, we first find an expression for the electric field in the periodic stack by substituting Eq. (3) into Eq. (1). By using Maxwell’s equations we can then find the magnetic field in the stack as well. Since both fields are transverse, the Maxwell saltus conditions prescribe that each is continuous over the interface. Our next task is to write these fields in terms of an incoming wave and a reflected wave in the linear medium. This is most conveniently done in the frequency domain. We thus Fourier transform the two fields, separate the incoming and reflected waves, and transform back. We then finally find the electric field in the linear medium,

$$
E(z, t) = \frac{A}{2} [\text{sech}(Bt)|C_1^i + C_2^i \tanh(Bt_i)| + \text{sech}(Bt)|C_1^r + C_2^r \tanh(Bt_r)| \exp(-i\omega_i t_i) + \text{c.c.}]
$$

(5)

where the center frequency $\omega_c = \omega_n + \delta + \Delta$ and the time arguments of the incoming and reflected waves are defined as $t_i = t - zn/c$ and $t_r = t + zn/c$ (for $z < 0$ only). The two coefficients $C_i^r, C_i^r$ are of order unity, whereas $C_2^r, C_2^r$ are an order of magnitude smaller. All four (complex) coefficients depend on the detailed properties of the Bloch functions in the periodic medium and are not written out here. Still, it is clear that Eq. (5) describes an incoming and reflected pulse with hyperbolic secant shapes and a temporal width of approximately $1/B$. The coefficients $C_2^r$ merely shift the pulses slightly and add a small asymmetric component to the phase.

The theory we presented above was derived for a strictly one-dimensional geometry. We have shown before, however, that our theory can also be used to describe the electromagnetic fields in waveguide environments in the single-mode approximation. Under this approximation, Eqs. (1)-(5) are still valid, but the total electric field in the waveguide is now found by multiplying the resulting fields in Eqs. (1) and (5) by the appropriate eigenmode of the waveguide. This simple procedure requires that the coefficient $\alpha_m$ be suitably redefined and assumes that the relevant waveguide mode is TE polarized. It should be noted that in waveguides it is the guide index $N$, rather than the refractive index, that is to be modulated periodically. This can be achieved by modulating the thickness of the guiding layer (see Fig. 1). Optical waveguides are particularly interesting as they appear to provide the most promising geometry to observe gap solitons experimentally. For this reason we apply the present theory to phenomena in optical waveguides. We take the following realistic parameters for a waveguide structure: $n_c = 1.0$, $n_f = 1.6$, $n_s = 1.46$, $h = 500$ nm, $\Delta h = 20$ nm, and $d = 355.4$ nm (see Fig. 1), where the $n$’s refer to the linear refractive indices. For radiation with a wavelength of 1.0653 $\mu$m, we find that $N = 1.49888$ and $\omega_m = 1.867 \times 10^4$ m/s/sec. As before, in line with current research in the field of nonlinear optics, we choose the guiding layer to be nonlinear with $n^{(3)} = 10^{-16}$ m$^2$/W. Also, we assume that the energy flux through the system is limited by the damage threshold of the nonlinear guiding material, which we take to be $S_d = 1$ GW/cm$^2$. This defines the maximum amplitude of the envelope function, which, in turn, defines an upper limit for the detuning $\delta$ [Eqs. (3) and (4)]. Based on these considerations, we take $\delta = -243.2 \times 10^9$/sec, so that $B_1 = 5.10/\mu$m, which implies that our soliton is approximately 0.2 mm wide [Eq. (3)]. In addition, we choose $\Delta = -\delta/16$, so that $B_2 = 1.28/\mu$m, and the incident pulse width $1/B = 8.2$ psec. The generated soliton has the same temporal width but travels at a velocity of $v_s = 2.38 \times 10^7$

![Fig. 1. Schematic of the waveguide geometry.](image-url)
1.00
0.75
0.50
0.25

Fig. 2. Time- and space-averaged electric-field energy densities as a function of position at $t = -15$ psec (dashed curve), $t = 2.5$ psec (solid curve), and $t = 20$ psec (dotted curve).

m/sec, or $v_s/(c/N) \approx 0.12$. We thus see that the soliton travels more than eight times slower than a pulse in a medium with the same average index would and is consequently eight times narrower (spatially). Our choice for $\delta$ and $\Delta$ gives a center frequency such that $\lambda = 1.0641 \mu m$. By choosing smaller values for the parameter $\Delta$ we can, in principle, make $v_s$ as small as we wish, thus clearly demonstrating the possibility for low-velocity energy transport.

Our results are illustrated in Fig. 2, which shows the normalized time- and space-averaged electric-field energy densities as a function of position for different times. In these calculations we have neglected the nonlinearity in the noncorrugated part of the waveguide. This nonlinearity causes a chirp that is proportional to the distance traveled. Thus in an actual experiment one would either inject the light close to the corrugated section or give the incoming pulse a chirp so as to cancel the effect of propagation through the uncorrugated part. The necessary calculations are straightforward but depend on the coupling geometry and are not given here. Figure 2 confirms our expectations: the incoming wave, coming from the left, reflects off the interface, launching a slowly traveling soliton in the periodic material. Note that, as required, the amplitude of the incoming wave is higher than that of the reflected wave. The reflection coefficient is small in this case, which is consistent with the low soliton velocity. It can, in fact, be shown that the reflectivity is of the order of $4v_s/c$, which would be the Fresnel expression in the limit of large refractive index difference. It should be noted that the theory can be scaled to accommodate the wavelength of choice. The period of the surface corrugation cannot be made arbitrarily small, however, thus setting a lower limit to the scale of the waveguide geometry. We note in passing that at much lower intensities, where the influence of the nonlinearity is negligible, the transmission into the corrugated part of the waveguide is small, as most of the frequency spectrum of the pulse overlaps the stop gap. The pulse is thus almost completely reflected, while mostly retaining its original shape.

We close by noting the similarity between the concentration of energy in these solitons, which locally burn themselves through the stop gap, and the concentration of energy that could arise if a defect were introduced into a linear periodic structure and a resulting localized state were excited. The solitons we describe can thus be thought of as self-localized light. Since the localization is self-induced it is a richer phenomenon than the linear analogue; in particular, the localized state can move through the material and could, for example, collide with other such localized states. We will investigate such matters in a future publication.

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