Power estimates for the launching of gap solitons in nonuniform gratings

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Abstract

Estimates are derived for the required launching intensity of gap solitons in nonuniform gratings, based on the linear resonance properties of the gratings. Such estimates provide a valuable tool for determining the feasibility of experimental work.

1. Introduction

Theoretical investigations of light propagation through nonlinear Bragg gratings predict the existence of solitary waves known as gap solitons [1]. A key property of gap solitons is their ability to traverse the grating at any speed between 0 and \( v_g \), the speed of light in the material. Recently Sankey et al. [2] observed all-optical switching in a nonlinear grating [3] mediated by stationary gap solitons. While their experiment demonstrated some of the CW properties of gap solitons, experimental confirmation of gap soliton propagation is still lacking. Such experiments are hampered by the high external powers required for gap soliton generation.

High powers are necessary because the spectrum of gap solitons lies within the band gap of the grating where, in the linear regime, no travelling wave solutions exist, and incident radiation is strongly reflected; the coupling of light into the gratings at these frequencies is thus inherently inefficient. The launching of gap solitons by incident pulses in a straightforward way is thus found to require prohibitive pulse energies, although the energy required for a gap soliton to exist in a grating structure once it is somehow launched is more modest [1].

In a previous paper, two of us [4] examined the complex reflection spectrum of nonuniform gratings. The zeros of this spectrum are of obvious interest in coupling light into the structure: At the zeros the reflection vanishes and, at least in the linear regime, all the light is coupled into the grating. For a uniform grating these zeros occur for real frequencies lying outside the photonic band gap. Though one cannot design a grating which has a real zero inside the photonic band gap, it was shown [4] that one can design a nonuniform grating with complex zeros, such that the real part lies within the photonic band gap. It was also shown that such zeros provide an efficient channel for coupling gap solitons into the grating. We note that the imaginary part of a complex frequency describes the exponential growth or decay of the input signal. Thus to utilise these complex frequency reflection zeros the incident light should be a pulse, with a leading edge that is exponentially growing. The simplest such pulse is hyperbolic secant shaped, on which we concentrate.
In this paper we present estimates for the intensity required to launch a gap soliton. These intensity estimates allow quick determination of the feasibility of possible experiments.

2. Gap solitons

The propagation of light within a periodic medium is here modelled using coupled mode theory. For a Bragg grating the only modes of interest are the forward and backward propagating modes whose slowly varying envelopes are denoted by \( F_+ \) and \( F_- \) respectively. In terms of these envelopes the electric field \( E \) can be expressed as

\[
E(t,z) = [F_+(t,z) \exp(+i k_0 z)] + F_-(t,z) \exp(-i k_0 z)] \exp(-i \omega_0 t) + \text{c.c.} \quad (1)
\]

Here \( k_0 = \pi/d \) is the Bragg resonance and \( \omega_0 \) is the associated frequency. The refractive index profile \( n(z) \) of the grating is given by

\[
n(z) = n + \Delta n \cos(2 k_0 z) \quad (2)
\]

The envelopes \( F_+ \) and \( F_- \) obey the following nonlinear coupled mode equations [3]:

\[
\begin{align*}
\frac{d F_+}{dz} + \frac{i}{v_b} \frac{d F_+}{dt} + \kappa F_- &= + 2 \Gamma_s |F_-|^2 F_+ + \Gamma_s |F_+|^2 F_- = 0, \\
\frac{d F_-}{dz} + \frac{i}{v_b} \frac{d F_-}{dt} + \kappa F_+ &= + 2 \Gamma_s |F_+|^2 F_- + \Gamma_s |F_-|^2 F_+ = 0,
\end{align*}
\]

where [5]

\[
\kappa = \frac{\pi \Delta n}{\lambda}, \quad \Gamma_s = \Gamma_x = \frac{4 \pi n}{\lambda Z n(2)}.
\]

The parameter \( \kappa \) measures the strength of the linear coupling between the modes and is proportional to the maximum refractive index \( \Delta n \) change in the grating. Nonlinear effects are described by \( \Gamma_s \) and \( \Gamma_x \) which represent self and cross-phase modulation respectively. In our geometry \( \Gamma_x = \Gamma_s \), although in other geometries this may not always be the case.

The solitary wave solutions to Eqs. (3) found by Aceves and Wabnitz [6], form a two-parameter family with parameters \( v \) the velocity of the gap soliton, and \( \Xi \) which determines the width and centre frequency of the gap soliton. These solutions are single peaked with a hyperbolic secant-like intensity profile. The velocity affects both the width and centre frequency but only to second order. As this paper concentrates on slow gap solitons, these velocity effects are neglected.

The energy, FWHM and the central frequency can all be defined in terms of the parameter \( \Xi \); at low velocities the energy of a gap soliton is proportional to

\[
Q = \int_{-\infty}^{+\infty} (|F_+|^2 + |F_-|^2) dz = \frac{4 \Xi}{3 \Gamma_x} \quad (5)
\]

For small values of \( \Xi \) the FWHM \( w \) of a gap soliton is approximately

\[
w \approx \frac{2}{\kappa \sin(\Xi)} \quad (6)
\]

while the frequency difference between the centre frequency of the gap soliton and the Bragg resonance \( \omega_0 \) is \( v_c \delta^g \) where

\[
\delta^g = \kappa \cos(\Xi) \quad (7)
\]

Eqs. (6) and (7) imply that

\[
\delta^g + 4/w^2 = \kappa^2 \quad (8)
\]

which describes an ellipse in the \( (\delta^g, 1/w) \) plane.

3. Linear grating theory

Linear gratings are characterised only by the parameter \( \kappa \) and the detuning \( \delta \), where

\[
\delta = \frac{\omega - \omega_0}{\nu_b} \quad (9)
\]

and where \( \delta \) is the deviation of the wavenumber from the Bragg wavenumber \( k_0 \) of the grating. Note that \( \delta \) can be real or complex depending on whether \( \omega \) is real or complex. This is in contrast to the (real) gap soliton parameter \( \delta^g \) defined by Eq. (7) which gives the detuning of the gap soliton and hence its centre frequency.

In the linear limit the envelope functions \( F_+ \) and \( F_- \) satisfy the linear coupled mode equations, i.e.
Eqs. (3) with \( I_x = I_y = 0 \). As the reflection spectrum is a CW property we can assume harmonic time dependence at a complex frequency \( \omega \).

For a uniform grating of length \( l \) the linear coupled mode equations can be solved exactly. This solution shows that the zeros in the reflection spectrum occur at real detunings \( \delta \) which are all outside the photonic bandgap. However the efficient launching of gap solitons requires reflection zeros whose real part lies inside the band gap. As this is not the case for the zeros of a uniform grating we need to use nonuniform gratings to launch gap solitons this way. The simplest such nonuniform grating is the step grating, which is shown in Fig. 1. The coupled mode equations can be solved exactly by considering the step grating as composed of two uniform gratings and matching the solutions at the interface; in fact the concatenation of any number of uniform gratings can be solved by this approach. This provides an exact expression for the complex amplitude reflection function \( r(\delta) \). Our numerical work has shown that step gratings can have complex zeros whose real part lie inside the band gap, and so are suitable for coupling light from pulses into the grating.

Consider such a zero with a complex detuning \( \delta = \delta_r + i\delta_i \) and associated complex frequency \( \omega \) given by Eq. (9). An input pulse which is "tuned" to this zero has the form:

\[
\mathcal{F}_+(t, 0) = A \text{sech}(v_g \delta_i t) \exp(-i v_g \delta_r t).
\]

The temporal width \( T \) of the input pulse is approximately given by

\[
T = \frac{1}{\delta_i v_g}.
\]

Thus the imaginary part of the complex zero is linked to the pulse width, while \( \delta_r \) is related to the pulse's frequency through Eq. (9). For such a hyperbolic se-cant shaped pulse, the trailing edge does not have the correct complex detuning and so it is reflected by the grating. Thus any gap solitons launched with such a pulse must be formed by the leading edge.

Furthermore each zero in the complex frequency plane corresponds to a resonance inside the grating. Fig. 2 shows the electric field profile for a complex zero with detuning \( \delta = 19.7708 + i 0.50605 \text{ cm}^{-1} \). The parameters of the step grating are: \( l_1 = 0.1585 \text{ cm}, \ l_2 = 3.5 \text{ cm}, \ k_1 = 16 \text{ cm}^{-1} \) and \( k_2 = 20 \text{ cm}^{-1} \). Most of the light is confined to the front piece of the grating leading to a strong resonance effect. This can be clearly seen from Fig. 2 which shows that the peak intensity inside the grating is over a factor four higher than the intensity outside, facilitating gap soliton formation.

Since the step grating has a linear waveform (as shown in Fig. 2) associated with the resonance that is similar to a gap soliton it is reasonable to assume that as the energy content of this linear resonance increase this wavefunction can evolve into a gap soliton which is born out by our numerical simulations. For this process to occur efficiently the width and detuning of the linear resonance must lie on the ellipse given by Eq. (8) which describes the relationship between the width and detuning of the gap soliton. This postulated formation process implies that the gap soliton formed has approximately the same width and frequency as the linear resonance of the grating. It enables us to determine from the linear resonance the value of the

- Fig. 2. Intensity profile of the linear waveform associated with a complex reflection zero of the step grating described in the text. The solid line shows the total intensity. The dotted and dashed lines represent that associated with the forward and backward propagating fields, respectively. Note that, as required at a reflection zero, the latter vanishes at the front of the grating. The waveform is sharply peaked with a peak intensity inside the grating which is over four times higher than the incident intensity.

- Fig. 1. Schematic of a step grating. The front grating has a length \( l_1 \) and a strength \( k_1 \). The main grating has a length \( l_2 \) and a strength \( k_2 \).
soliton parameter $\Xi$, and thus the energy of the gap soliton. As the velocity of the soliton is weakly coupled to the shape, this method gives no estimate for the gap soliton velocity.

The question of which step gratings are best for the launching of gap solitons depends on the available parameter space. Experimentally the maximum length of a grating is currently about 4 cm [7] and for gratings written in optical fibres the typical strength is about $\kappa = 300 \text{ cm}^{-1}$ at a wavelength of 1.06 $\mu$m. Another constraint form the available pulse lengths of the light source, which determines the possible values that the imaginary part of the complex zero can take.

In addition to the upper bounds for the length and strength of the gratings, there are also lower limits. These are due to the fact that if the grating is too weak then there are no complex zeros lying above the band gap, and thus the grating is of no use in launching gap solitons. We then take a sample of feasible gratings and examine their reflection spectrum. If they have a complex zero whose associated linear waveform lies on the ellipse given by Eq. (8) the step grating can then be used to launch gap solitons.

4. Power requirements

In this section we assume that the grating structure has a complex zero at $\delta = \delta_r + i\delta_i$. The width $w$ and frequency $\nu_g \delta_r$ of the associated linear resonance are assumed to lie on the ellipse given by Eq. (8); in addition we assume that $\delta_i$ is compatible with the available pulse lengths. The input pulse is then given by Eq. (10), where $A^2$ is proportional to the peak input power. Since the grating is designed to have a reflection zero at this frequency we can assume that all the energy in the leading edge of the pulse is used to form the gap soliton and so nothing is reflected. Clearly the energy in the front half of the pulse must exceed than the energy in the gap soliton that we desire to form. This leads a minimum value for $A^2$:

$$A^2 > \frac{4\delta_r \Xi}{3 \Gamma_x}.$$  \hspace{1cm} (12)

Inverting Eq. (7) and expressing $\Gamma_x$ in terms of the nonlinear refractive index $n^{(2)}$ as given in Eq. (4), we can express the above equation in terms of the peak intensity $P_p$:

$$P_p > \frac{2\lambda}{3\pi n^{(2)} T \nu_g} \cos^{-1}\left(\frac{\delta_r}{\kappa}\right),$$  \hspace{1cm} (13)

where $\nu_g$ is the pulse length given by Eq. (11), and $\lambda$ the free space wavelength. Substituting the values for glass [8], and defining $T'$ in terms of 100 ps we then get

$$P_p > 1.6 \times 10^7 \frac{\lambda}{T'} \cos^{-1}\left(\frac{\delta_r \lambda}{\pi \Delta n T'}\right) \text{GW/cm}^2.$$  \hspace{1cm} (14)

In the following equations we evaluate the formulae using parameters suitable for optical fibre experiments.

Eq. (13) suggests that the intensity requirements can be made arbitrarily low by either: (a) Increasing $T$ or (b) taking $\delta_r$ arbitrarily close to $\kappa$. If we assume that we have a fixed laser source which can only produce pulses of certain lengths, case (a) is unrealistic, as we are unable to increase $T$. In addition we have found that the effect of increasing $T$ is to lengthen the front grating, beyond reasonable practical limit.

Case (b) arises from the fact that when the detuning of the gap soliton approaches the band edge $\Xi = 0$ the energy in a gap soliton approaches zero. However a closer examination of the solutions shows that the width of the gap soliton in this case diverges. Clearly we are unable to launch a gap soliton which is longer than our grating. This maximum width requirement equates to a minimum $\Xi$ through Eq. (6) which then identifies a maximum $\delta_r$. When substituted into Eq. (13) leads to the minimum intensity requirement:

$$P_p > \frac{4\lambda}{3\pi n^{(2)} T \nu_g \kappa l_2} = 0.01 \frac{\lambda^2}{T' \Delta n l_2^2} \text{GW/cm}^2,$$  \hspace{1cm} (15)

where $l_2$ is the length of the main grating, $l_2'$ is the length in millimetres, $\lambda'$ is the wavelength in micrometres and $T'$ is the length of the pulse measured in 100 ps.

An additional restriction we place concurrently on the allowed parameters is that a significant fraction of the incident light is reflected at low intensities, ensuring a clear distinction between the low- and high-intensity regimes. Here we require that $\delta_r + 0.45 \delta_i < \kappa$. For a hyperbolic secant shaped pulse this corresponds to 80% of the incident light being within the band gap of the main grating, ensuring that in the linear regime at least 80% of the incoming light is reflected. Assuming that the pulse length is fixed, this
then puts a different limit on the maximum value of $\delta_r$. The minimum intensity needed is now

$$P_p > \frac{2}{\sqrt{5} \pi n^{(2)} T^3/2 v_p^{3/2} K_{1/2}} \frac{\lambda}{T^3 \Delta n} \text{ GW/cm}^2.$$  \hspace{1cm} (16)

Eqs. (15) and (16) present minimum intensity requirements for the generation of gap solitons assuming a particular grating design. In contrast Eq. (13) gives the minimum intensity requirement to launch a specific gap soliton and this intensity must be higher than the intensity given by both Eqs. (15) and (16) to ensure that all the restrictions are met.

5. Implementation

To show the validity of Eqs. (13)–(16) we present the results of numerical simulations. The step grating used for these results has a slightly different design from the one shown in Fig. 1. We have made the grating symmetric by addition another grating to the end of the grating shown in Fig. 1. This additional grating, which is identical to the front grating, couples light out of the grating, just as the front grating facilitates the coupling of light into the grating. The back grating does not affect the formation of gap solitons to any noticeable extent as the middle grating is sufficiently long that, in the linear regime an exponentially small amount of energy enters the back grating. We use the same step grating and resonance as described in Section 3 with the centre of the Bragg resonance at $\lambda = 1.06 \mu m$ and $n^{(2)} = 3 \times 10^{-20} \text{ cm}^2/\text{W}$ [8]. The input pulse is a hyperbolic secant shaped pulse with a FWHM of 260 ps. For this particular step gratings the minimum intensity requirements given by Eqs. (15) and (16) are 1.0 GW/cm$^2$ and 5.7 GW/cm$^2$ respectively. We must take the higher of the two as our minimum intensity. This means if the intensity given by Eq. (13) is greater than 5.7 GW/cm$^2$ then the launched gap soliton has a length smaller than 3.5 cm and that for a weak 260 ps input pulse at least 80% of the energy is reflected.

For this grating a suitable reflection zero and hence pulse detuning occurs at $\delta = 19.7708 + 10.50605 \text{ cm}^{-1}$ and the width of the linear resonance is such that the zero lies on the ellipse given by Eq. (8). The resulting coupled mode equations were solved numerically [9] and the results are discussed below.

Fig. 3 shows the linear response of the grating. The solid line shows the input pulse, the dotted the reflected pulse and the dashed line shows the transmitted pulse. Note that there is a noticeable time delay between the input and the reflected pulse. This large Goos-Hänchen shift arises as no light is reflected from the leading edge of the pulse whose complex frequency matches a zero in the reflection spectrum.

The basic equation for the intensity required [Eq. (13)] gives the value of 5.7 GW/cm$^2$ for the formation of a gap soliton. Our numerical simulations show that for incoming pulses with a peak intensity of about 6 GW/cm$^2$ low-intensity gap solitons are generated, confirming the validity of our approach. However because of the nonlinear nature of this process the intensity of the gap soliton can be significantly increased by only slightly increasing the incident intensity. We therefore show in Fig. 4 the results for an incident pulse with a peak intensity of 7.5 GW/cm$^2$. Fig. 4 shows that a gap soliton forms, which moves through the grating at about $v_g/6$ and exits the grating after about 9 transit times (where one transit time equals $L/v_g = 191 \text{ ps}$). The effectiveness of our procedure can be seen in Fig. 5 which shows the results of the numerical simulation for a uniform grating. Although the peak input intensity is no less than 10 GW/cm$^2$, in contrast to the step grating simulation at a lower intensity here nearly all the light is reflected.
Fig. 4. Incoming (solid line), reflected (dotted line) and transmitted (dashed) energy as a function of time at high intensity for the step grating discussed in the text. The peak in the transmitted intensity after about 2 ns indicates that a gap soliton has been generated.

Fig. 5. Incoming (solid line), reflected (dotted line) and transmitted (dashed) energy as a function of time for a uniform grating at the same intensity as in Fig 4. The absence of any substantial transmitted energy indicates that no gap soliton has been generated.

6. Discussion

Eqs. (13), (15), and (16) give a rough estimate for the intensity required to create gap solitons. The assumption that no light is reflected is not generally true, as once there is light inside the grating the reflection properties of the grating change due to the nonlinearity and we no longer have a zero at the designed frequency. A fuller treatment would take into account the effect the nonlinearity has on the position of the zeros. These effects are most pronounced for gratings with a strong resonance as these have the highest intensities inside the gratings. These resonances correspond to zeros with a small imaginary part. In such cases the resonance quickly detunes itself from the incoming pulse and so little subsequent energy enters the grating and hence our estimates are not valid.

All three equations predict that the peak intensity required to launch a gap soliton can be made arbitrarily small by increasing the pulse length. This results from our postulated formation process which only depends on the total energy in the input pulse and not the intensity. However in practice there are three main reasons why the equations are not valid in this regime. The first is that long pulses require zeros with very small imaginary part which only occurs for extremely long and hence impractical gratings. The second is that the longer the pulse the more sensitive the corresponding zero is to the precise shape of the pulse. The last effect is that as energy enters the grating the position of the zero moves and so the grating “detunes” itself away from the driving field. Theoretically if we designed the pulse so that its instantaneous frequency matched the position of the zero, then all the light would be coupled into the grating. Such custom designed pulses could have an arbitrarily small peak intensity. Although perfect coupling is probably unattainable it is likely that some improvement in the coupling could be achieved by using non hyperbolic secant-like pulses.

In deriving the equations for the minimum intensity requirements we have assumed that we can always find a suitable nonuniform grating and in particular a suitable step grating. This assumption is in principle valid as gratings can be designed by use of inverse scattering [10] to have a zero at any required complex frequency. However in general such gratings are not simple step gratings. Our numerical work suggests that a suitable step grating can always be found to launch a particular gap soliton. We stress that the validity of the above formulae depends on choosing the correct grating design.

7. Conclusion

In this paper we have presented for the first time, estimates of the intensity required for launching gap solitons. Such estimates are important as they provide a quick means for determining the feasibility of possible experiments. These estimates are based on step gratings with linear resonances, leading to field structures which evolve into gap solitons at suitably high intensities. Although we have only considered step gratings in any detail the basic formula derived Eq. (13)
depends only on the fact that we have created a resonance structure. Thus, it is valid for any nonuniform grating, which has complex zeros with the right properties. Eqs. (15) and (16) describe the gap solitons that we are interested in generating. These equations reflect the use of step gratings and so are not generally applicable. However for different nonuniform gratings we would expect similar criteria.

This linear resonance method for the launching of gap solitons also gives an estimate for the gap soliton parameter \( \Xi \), (which determines the height, width and centre frequency of the gap soliton) as it is determined by the width of the linear resonance. This then enables us to work backward to determine the most appropriate step grating for the launching of a particular gap soliton. As the velocity of the gap soliton is only weakly coupled to the width we are unable at present to predict the velocity of the resultant gap soliton.

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