A cluster expansion approach to modelling single-channel bit string propagation in optical fibers

V.B. Deyirmenjian¹, J.E. Sipe
Department of Physics, University of Toronto, 60 St. George St., Toronto, Ont., Canada M5S 1A7
Received 28 July 1998; accepted 1 September 1998

Abstract

A method for determining the nonlinear propagation of a wavetrain of non-return to zero (NRZ) pulses in an optical fiber is presented. The evolution of a bit string is calculated by propagating the fundamental pulses individually without interpulse effects and then systematically adding corrections due to two, three, and higher order interacting pulses. This approach can be used to reduce the computational time for propagating a long wavetrain or a set of many similar bit strings.

The analysis of pulse propagation in optical fibers is of obvious scientific and technological interest [1]. Recently, Lazaridou, Debarge, and Gallion introduced a scheme for solving the linear propagation equation [2], employing a basis set of chirped Gauss–Hermite functions for which the linear evolution is known analytically. One would like to apply such a scheme to nonlinear propagation. But this is not possible, since there are no analytical results for the nonlinear evolution of arbitrarily shaped pulses. Numerical analysis at some level is required, but the goal is to keep the analysis as simple as possible.

In this paper, we describe an approximate procedure for numerically propagating a single-channel non-return to zero (NRZ) wavetrain subject to dispersion, loss, and nonlinearity in an optical fiber. The initial bit string is described as the sum of fundamental pulses. A “fundamental pulse” is an isolated group of neighboring on bits, such as a single on bit or a finite number of neighboring on bits. For example, the fundamental pulses of the string \(1011011100101\) are a one-on, followed by a two-on, a three-on, and then two one-ones. The nonlinear propagation equation is solved numerically for each of these pulses in the absence of interpulse effects. Corrections due to the interactions between the pulses can then be determined and added as required.

An input string of \(N\) fundamental pulses at \(z = 0\) gives a total field

\[
u(0, \tau) = \sum_{i} u_i(0, \tau),
\]

where the subscript \(i\) runs from 1 to \(N\) and indicates both the number of on bits in the fundamental pulse, and its position in time. At a distance \(z\) down the fiber the total field \(u(z, \tau)\) is expanded as

\[
u(z, \tau) = \sum_{i} u_i(z, \tau) + \sum_{ij} \phi_{ij}(z, \tau) + \ldots,
\]

where \(u_i(z, \tau)\), found numerically, is the field to which \(u_i(0, \tau)\) would evolve if no other pulses were present; \(\phi_{ij}(z, \tau)\) is the correction due to the interaction between two fundamental pulses \(i\) and \(j\); \(\phi_{ijk}(z, \tau)\) is the three pulse correction involving \(i, j, \) and \(k\), and so on. As usual, \(\tau = t - z/\nu_g\) is the local time coordinate, with \(\nu_g\) the group velocity. The \(\phi\) functions are developed by considering the interactions of \(n\) pulses for \(n = 2, 3, \ldots, N - 1\).

We numerically evolve the field \(u_{ij}(0, \tau) = u_i(0, \tau) + u_j(0, \tau)\) to find \(u_{ij}(z, \tau)\); \(\phi_{ij}\) is then defined according to

\[
\phi_{ij} = u_{ij} - u_i - u_j.
\]
Fig. 1. Power of the field $u$ of the string (1011011100101) at (a) $z = L$ and (b) $z = 3L$ with $\tau_s = 10$ GHz and $L = 120$ km, the length of one fiber link. In these figures, the solid curves indicate the results of propagation subject to dispersion, loss, and nonlinearity. The dashed and dotted lines denote the powers of $s^{(1)}$ and $s^{(2)}$, respectively. In (a), the curve labelled $u^{(L)}(L, \tau)$ represents the power of the input string after dispersive evolution. The short dash-dotted line is the power of the input wavetrain at $z = 0$. 


Similarly, the three pulse string \( u_{ijk}(0, \tau) = u_i(0, \tau) + u_j(0, \tau) + u_k(0, \tau) \) can be numerically evolved to find \( u_{ijk}(z, \tau) \); \( \phi_{ijk} \) is then defined according to

\[
\phi_{ijk} = u_{ijk} - u_i - u_j - u_k - \phi_{ij} - \phi_{ik} - \phi_{jk}.
\]

and so on. The output field then follows from the expansion (2); the scheme is similar to the cluster expansion in statistical mechanics [3]. The highest order correction needed in practice depends on the system parameters, but at any level only a few numerical calculations of the relevant \( \phi' \) are required, especially since beyond few-nearest-neighbors the \( \phi' \) are found to essentially vanish.

For the conditions discussed below and a bit rate of \( \tau_s = 10 \, \text{GHz} \), two pulse corrections \( (\phi_{ij}) \) are sufficient to construct a good approximation of the exact result. For linear propagation, of course, all of the \( \phi' \) vanish.

We model the propagation using the one dimensional nonlinear Schrödinger equation (NSE), modified to include loss and third order dispersion [6],

\[
\frac{\partial u(z, \tau)}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 u(z, \tau)}{\partial \tau^2} + \frac{i \beta_3}{6} \frac{\partial^3 u(z, \tau)}{\partial \tau^3} - \gamma |u(z, \tau)|^2 u(z, \tau) - \frac{\alpha}{2} u(z, \tau),
\]

where \( \beta_2 \) (\( \beta_3 \)) is the second (third) order dispersion, \( \gamma \) is the nonlinear coefficient, and \( \alpha \) is the fiber loss; higher order corrections could easily be included. The input bit profile is modelled by passing rectangular pulses through a Bessel filter of order 20 and a 3 dB bandwidth of 2 \( \tau_s \) [4,5]. The exact solution \( u \) and the basic units \( u_i, u_{ij}, u_{ijk} \) are propagated by solving (5) via the usual split-step Fourier method [6]. A suitable number of zero bits separates the input string from the ends of the Fourier

---

Fig. 2. Dispersion compensation of the exact \( u \) and the approximate solutions \( s^{(1)} \) and \( s^{(2)} \) after nonlinear propagation of the string (1011011100101) through three fiber links with dispersion and loss. The dash-dotted line shows the compensated linearly evolved result at \( z = 3L \), which reproduces the input wavetrain.
transmission compensation, we denote the field dispersion compensation. After an assumed exact dispersion correction, we can ignore with a negligible decrease in accuracy. The inclusion of higher order corrections results in many similar bit strings. For bit rates up to about 10 GHz, the inclusion of two pulse corrections gives a good estimate of the exact result, and of the dispersion compensated signal. We hope to develop an analogous approach for multichannel problems.

Acknowledgements

This research was supported by the Natural Sciences and Engineering Research Council of Canada and Photonics Research Ontario.

References