NONLINEAR S-POLARIZED SURFACE PLASMON POLARITONS

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We have examined the dispersion relations for s-polarized surface plasmon polaritons, guided by (a) the interface between a semi-infinite metal and dielectric medium, and (b) a metal film bounded by semi-infinite dielectric media for situations in which one or more of the dielectric media are characterized by an intensity-dependent refractive index. We found that s-polarized waves satisfy the dispersion relations for very thin metal films bounded by nonlinear dielectric media. These waves exist only for power levels above a threshold that depends on the material parameters. We also comment on the experimental feasibility of observing these waves.

INTRODUCTION

The variety of waves guided by single or multiple interfaces changes when one or more of the guiding media are characterized by an intensity-dependent refractive index. The boundary between two linear dielectric media cannot support an s-polarized surface plasmon. However, if one or both of the media exhibit an intensity-dependent refractive index, it has been predicted that an s-polarized surface plasmon should exist at high powers. It has also been shown that new s- and p-polarized waves exhibiting power thresholds should occur for dielectric films bounded by nonlinear media. For metal films, new waves have also been predicted for the p-polarized case. However, the case of s-polarized surface plasmon polaritons guided by virtue of nonlinear media has not yet been reported. The results of a cursory search involving large values of \(|\varepsilon_m|\) proved negative. In this paper we examine this problem in more detail and find solutions that correspond to s-polarized waves over restricted ranges of material parameters and geometries.

DISPERSION RELATIONS

The dispersion relations for waves guided by an arbitrary film bounded on one or both sides by nonlinear media has been reported by us before. The field distributions in the cladding, film, and substrate are proportional to \(\exp(i(\mathbf{k} - \mathbf{w}t))\) and are given respectively by

\[
E_{cy}(x,z) = \frac{1}{2} \sqrt{\frac{q}{a_c}} \frac{g}{\cosh[k_q(z_s-z)]} e^{i(k_s x - \mathbf{w}t)} + c.c. \tag{1a}
\]

\[
E_{fy}(0,z) = E_{cy}(0,0) \times \left[ \cosh[k_s z] + \frac{3}{2} \tanh[k_q z] \sinh[k_s z] \right] \tag{1b}
\]

\[
E_{sy}(0,z) = E_{fy}(0,d) \cos[k_s (d-z)] \tag{1c}
\]

\[
E_{fy}(0,d) = \frac{2}{a_s} \frac{g}{\cosh[k_s (d-z)]} \tag{1d}
\]

for a film of thickness \(d\) and dielectric constant \(\varepsilon_m < 0\) bounded by semi-infinite media with refractive index given by \(n = a + \alpha n_2\) and \(a = n_0 + n_2\) for the cladding (upper medium) and substrate (lower medium) respectively, and with \(a_\perp = n_\perp^2 \varepsilon_s n_2 > 0\). The dispersion relation is

\[
\tanh(k_s z) = \frac{\alpha q \tanh[k_q z_s] + \alpha \tanh[k_q z_s]}{q^2 - \alpha q \tanh[k_q z_s] \tanh[k_q z_s]} \tag{2}
\]

Here \(q = \sqrt{\varepsilon_m}\), \(s = \sqrt{n_0^2 - \varepsilon_m}\) and \(\xi = \sqrt{\varepsilon_m - \varepsilon_s}\) where \(\mathbf{k_s}\) is the guided wave wavevector (\(k_s = \omega/c\)).
parameter \( z_1 \) is related to \( z_1 \) by Eq. (1d) and \( z_1 \) can be written either in terms of the total guided wave power or the Poynting vector at the cladding-film interface. In terms of the iterative calculations necessary for solving the dispersion relations, it is more convenient to define \( z_1 \) in terms of the Poynting vector \( S_0 \) at \( z = 0 \) as

\[
z_1 = \pm \frac{1}{k_{q}q} \arccosh \sqrt{\frac{a_{q}^2}{n_C n_{2C} n_{5}}}. \tag{3}
\]

The power per unit distance along the wavefront carried by the waves is given by:

\[
P_C = \frac{8q}{k_{q}n_C n_{2C}} \left[ 1 - \tanh(k_{q}x) \right]
\]

\[
P_f = \frac{8q}{k_{q}n_C n_{2C}} \frac{1}{\cosh^2(k_{q}x)} \left[ 1 - \frac{a_{q}^2}{x^2} \tanh(k_{q}x) \right] \left[ 1 - \frac{a_{q}^2}{x^2} \tanh(k_{q}x) \right] \sinh(2k_{q}x)
\]

\[
+ \frac{a_{q}^2}{x^2} \tanh(k_{q}x) \left( \cosh(2k_{q}x) - 1 \right)
\]

\[
P_s = \frac{8q}{k_{q}n_s n_{2s}} \left[ 1 - \tanh(k_{q}x) \right]. \tag{4c}
\]

**NUMERICAL RESULTS**

Our numerical calculations were based in a general way on the liquid crystal MBBA \((n = 1.55, n_2 = 10^{-1} \text{ m}^2/\text{W} \text{ in the blue-green region of the visible spectrum})\). This material is of particular interest because \( \Delta n_{\text{max}} \), the maximum change in refractive index which can be produced optically, is large, of the order of 0.1.

\( S \)-polarized solutions were found for very thin metal films bounded on both sides by nonlinear media characterized by positive values of \( n_2 \). The results are summarized in Figs. 1 through 4. The variation in guided wave power with effective index \( \beta \) is shown in Fig. 1 for a few metal film thicknesses and for identical nonlinear media bounding the metal film \((\varepsilon_m = -10)\). There is a definite power threshold at which the solutions first appear and that increases with film thickness. Note that for reasonable metal film thicknesses of 0.01 \( \mu \text{m} \) or more, the threshold value of \( \beta \) corresponds to a maximum index change in the bounding media in excess of 0.1 and is therefore beyond the available material limits. The field distributions, Fig. 2a, exhibit symmetric maxima in both bounding media, and these maxima move toward the metal film with increasing power.

The solutions are sensitive to the value of \( \varepsilon_m \). As \( \varepsilon_m \) decreases, that is \( |\varepsilon_m| \) increases, both the threshold power and minimum effective index increase rapidly. For example, for \( \varepsilon_m = -40 \), these values are 374 W/m and 2.05 respectively, which should be compared to 88.6 W/m and 1.61 for \( \varepsilon_m = -10 \) (\( d = 0.005 \mu \text{m} \)). This explains why the initial search\(^1\) at \( \varepsilon_m = -1000 \) failed. Unfortunately, for most real metals, this range of useful values for \( \varepsilon_m \) is usually characterized by imaginary components that result in high propagation losses.

Wave solutions also exist when the bounding media have different optical properties. For example, when the value of one of the refractive indices of the bounding media is changed, the solutions persist and the field distributions become asymmetric. Two examples are shown in Figs. 2b and 2c. Note that for large enough \( |n_0-n_1| \), fields are obtained with maxima in only one medium.

We also investigated the effect of \( n_2 \neq n_2 \). The results for the guided wave power versus \( \beta \) are shown in Fig. 1 and the corresponding field distributions in Fig 4. As the nonlinearity in the substrate is reduced, the threshold guided wave power increases correspondingly. Furthermore, the field distributions become distorted with progressively more of the power carried by the medium with the smaller nonlinearity.

These last results have bearing on the case when \( n_2 = 0 \) in one of the bounding media. From Figs. 3 and 4, the threshold power diverges in the limit \( n_2 = 0 \) and the fields are localized in the weakly nonlinear medium, that is, there are no physically realistic solutions. This was verified numerically with a

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**Fig. 1:** Guided wave power (mW/mm) versus effective index \( \beta \) for \( n_C = n_s = 1.55, \varepsilon_m = -10 \), and \( n_2C = n_2s = 10^{-4} \text{ m}^2/\text{W} \text{ for } d = 0.001 \mu \text{m} \) (solid line), \( d = 0.005 \mu \text{m} \) (dashed line), \( d = 0.010 \mu \text{m} \) (dotted line) and \( d = 0.015 \mu \text{m} \) (dash-dotted line).
s-polarized surface polaritons also do not exist for wave propagation along the interface between nonlinear semi-infinite dielectric and semi-infinite metallic media. This can be proven directly from the dispersion relation that is especially simple for this case, namely

$$\tanh(kq,_{s}) = \frac{\kappa}{q}.$$  

Because $$\kappa^2 = q^2 + n_s^2 = \kappa^2 + \kappa_m$$ for this case, $$\kappa > q,$$ hence $$\kappa/q$$ is always larger than unity and Eq. (5) can never be satisfied. Therefore s-polarized waves are not possible in this case. This helps explain why s-polarized polaritons in the thin metal film geometry exist only for reasonably small $$|\kappa_m|$$ and d. If the effective skin depth of the metal is on the order of d, the geometry essentially separates into two independent metal-dielectric interfaces at which s-polarized polaritons do not exist even at high powers. It is of course precisely because p-polarized surface polaritons do exist at such independent interfaces.
that p-polarized polaritons in the thin metal film geometry exist over much larger ranges of $|\varepsilon_m|$ and $d$.

In summary, we found that s-polarized surface polaritons can exist for thin metal films bounded by nonlinear media with positive nonlinearities, but not for any other planar geometry involving metals. These waves all have power thresholds. The combination of the very thin metal thickness required, the losses usually associated with surface plasmons for small $|\varepsilon_m|$, and the large changes in refractive index required will probably make such waves difficult to observed experimentally.

**Fig. 3**: Guided wave power (mW/mm) versus effective index $\beta$ for $n_a = n_b = 1.55$, $d = 0.005$ nm, $\varepsilon_a = -10$ and $n_{2c} = 10^{-9}$ m$^2$/W with $n_{2S} = 10^{-10}$ m$^2$/W (solid line), $n_{2S} = 10^{-11}$ m$^2$/W (dashed line) and $n_{2S} = 10^{-12}$ m$^2$/W (dotted line).

**Fig. 4**: Field distributions with increasing $\beta$ for $n_c = n_b = 1.55$, $d = 0.005$ nm, $\varepsilon_a = -10$ and $n_{2c} = 10^{-9}$ m$^2$/W for (a) $n_{2S} = 10^{-9}$ m$^2$/W, (b) $n_{2S} = 10^{-10}$ m$^2$/W and (c) $n_{2S} = 10^{-11}$ m$^2$/W.

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