## Period doubling of a femtosecond Ti:sapphire laser by total mode locking

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Period doubling of an 84-MHz repetition-rate Kerr-lens mode-locked Ti:sapphire laser operated at 830 nm and producing  $\sim$ 300 mW of average power has been observed and explained in terms of total mode locking of TEM<sub>00</sub> and TEM<sub>01</sub> modes in an effective confocal cavity. This configuration leads to a spatial sweeping action of a single-peaked pulse at 42 MHz. Period tripling and quadrupling is observed for certain cavity configurations. © 1998 Optical Society of America

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The Ti:sapphire laser has been a cornerstone of ultrafast science since the first demonstration of Kerr-lens mode locking (KLM) by Spence  $et \ al.^1$  Femtosecond pulses are now routinely generated over a tuning range of 700-1000 nm in X and Z cavities at a repetition rate of 80-85 MHz. Because these lasers are pumped by focused Gaussian beams from argon-ion or diode-pumped solid-state lasers, the gain volume waist is small and typically favors only a  $TEM_{00}$ oscillator mode. The KLM mechanism, which has been studied extensively both theoretically and experimentally,<sup>2</sup> uses self-focusing in the gain medium and requires careful alignment of the cavity mode with respect to the gain volume. Because of this and the nonlinear mechanisms required for passive mode-locked operation, it is not surprising that the Ti:sapphire laser exhibits a variety of interesting pulse dynamics, such as beam breakup,<sup>3</sup> self-Q switching,<sup>4</sup> and multiple pulsing.<sup>5</sup> Here we report the observation of period doubling of an 84-MHz KLM X-cavity Ti:sapphire laser that is due to locking of the  $TEM_{00}$  and  $TEM_{01}$ transverse spatial modes in addition to the longitudinal modes of the cavity. Such operation occurs near the stability edge of the region for normal mode-locked operation where the discrimination between continuous wave and mode-locked operation is greatest and where the X cavity is virtually equivalent to a simple confocal Fabry-Perot cavity.<sup>6</sup> The  $TEM_{01}$  mode frequencies then occur exactly between the  $TEM_{00}$  longitudinal mode frequencies. In such a case, as will be seen below, the locked superposition of the  $TEM_{00}$  and TEM<sub>01</sub> modes results in spatially single-peaked output pulses with a repetition rate of 84 MHz but that sweep back and forth transversely in space at 42 MHz, the intertransverse mode frequency. Transverse mode locking was discussed originally in the context of gas lasers by Auston<sup>7</sup> and Smith.<sup>8</sup> It should be noted that this type of period doubling differs from other kinds of period doubling, quasi-periodicity, and chaos-related instabilities, which are manifested mainly in the time domain.9,10

The laser consists of a commonly used X cavity<sup>6</sup> with two flat mirrors, a 20% output coupler  $M_1$ , a high reflector  $M_2$ , and two 10-cm-radius spherical mirrors  $M_3$  (prism side) and  $M_4$  (pump side) [unfolded version in Fig. 3(b)]. The spherical mirrors are lo-

cated approximately 5 cm away from the end faces of a Brewster-angled, 8-mm Ti:sapphire crystal pumped by 4.5–5 W of power from an argon-ion laser operating on all lines; the laser output wavelength is 830 nm. Two LaK-21 prisms are used for group-velocity dispersion compensation, enabling pulses of 80-fs width to be generated. The X-cavity geometry, Brewsterangled rod, and prisms define a preferential pulse polarization direction [taken to define the (horizontal) x axis, with the y axis being the vertical direction] and also produce the usual small degree of beam astigmatism. Figure 1 illustrates the temporal and spatial characteristics of the output pulse trains for normal (N) and period-doubled (PD) operation. The pulse trains are temporally monitored by a smallsized (<1 mm across), <2-ns rise-time Si photodiode coupled to a 400-MHz oscilloscope; the detector is located 50 cm from mirror M<sub>1</sub>. The oscilloscope traces clearly indicate the PD- and N-type laser operation but of themselves do not indicate the origin of this difference. However, when the PD and N beams are imaged by a CCD camera it is seen that N operation is associated with a near-Gaussian beam profile (apart from the small amount of astigmatism), whereas PD operation yields a two-lobed beam. However, the photodiode output is a function of the overlap of the approximately 700- $\mu$ m-diameter laser spot with



Fig. 1. (a) Left to right, oscilloscope trace of laser output in normal 84-MHz operation, overlap of the  $\text{TEM}_{00}$  mode with the Si photodiode, and image of the spatial mode profile recorded with a CCD detector. (b) Same as (a), except for period-doubled operation.

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the detector area. Indeed, by laterally translating the detector area along the y axis while the laser is in PD mode, we could alter the trace from a 42-MHz train [Fig. 1(b)] when the beam minimally overlaps the detector area, to an equal-amplitude train of 84-MHz pulses with the detector centered on the beam (cavity) axis, to another 42-MHz train when the beam minimally overlaps the other side of the detector. Both time-resolved and spatially resolved images can be explained if the output pulses for PD operation sweep back and forth along the y axis at 42 MHz. As the center of Fig. 1 indicates, the various degrees of overlap of successive pulses with the Si photodetector lead to 42-MHz modulation.

One achieves the PD behavior observed in Fig. 1 by moving curved mirror  $M_3$  away from the optimum (maximum average output power) position along the cavity axis by ~100  $\mu$ m. However, for starting purposes it is sometimes also necessary to tilt  $M_2$  or  $M_1$  by <2 mrad, to change the pump power or to displace the Ti:sapphire rod by 10–100  $\mu$ m. Figure 2 illustrates how the output power varies as a function of  $M_3$  displacement. PD operation as well as bistability with hysteresis is observed over a 60- $\mu$ m range of  $M_3$ , with the PD mode having a 2–5% lower average power than the N mode. PD operation can endure for as little as 30 s or as long as 1 h, depending on power level and cavity geometry. However, PD has never been observed when the laser output power is >350 mW.

We attribute the PD operation and beam sweeping effect to a total mode locking of the longitudinal modes and (mainly) the  $TEM_{00}$  and  $TEM_{01}$ spatial modes. Auston<sup>7</sup> has outlined the basic conditions for this type of mode locking to occur and indeed considered the general case of a superposition of phaselocked TEM<sub>0n</sub> (n = 0, 1, 2, 3...) modes. The Hermite-Gaussian modes are identical to the wave functions of a quantum-mechanical harmonic oscillator. An oscillating wave packet representing a classical state is obtained for a coherent superposition of a large number of modes whose amplitudes have a Poisson distribution. By analogy, the laser output field pattern consists of a Gaussian-like beam that moves back and forth in the transverse plane with simple harmonic motion at transverse mode spacing  $\Omega$ . The transverse mode spacing is not necessarily the same (it is the same only if the cavity is plano-plano) as the longitudinal mode spacing because the different transverse modes experience different longitudinal phase shifts in a cavity. For Hermite-Gaussian  $TEM_{mn}$  modes the complex field amplitude changes by a phase factor  $\exp[i(m+n)\theta]$  relative to the TEM<sub>00</sub> mode, where  $\theta$ can be obtained from, e.g., the ABCD matrix for the cavity. Because of the Kerr lens in the mode-locked Ti:sapphire laser a simple cold resonator cannot be used to approach the mode dynamics because one must either find a self-consistent ABCD matrix<sup>11</sup> or propagate the spatial profile by a nonlinear beam propa-gation method.<sup>12</sup> The phase factor is related to the cavity Guoy phase shift including a contribution from the Kerr lens. In general, the mode frequency  $\omega_{mnq}$  for a resonator of optical length L with cavity longitudinal frequency  $\omega_c = \pi c/L$  is

$$\omega_{mnq} = q \,\omega_c + (m+n+1)\omega_c(\theta/2\pi)$$
  
=  $q \,\omega_c + (m+n)\Omega + \text{const.},$  (1)

where q is the index for the longitudinal mode, m and n are the indices for the transverse modes, and  $\Omega \equiv \omega_c(\theta/2\pi)$  is the frequency separation of consecutive transverse modes. When the laser operates in a single Hermite–Gaussian mode,  $H_{mn}(x, y)$ , the standard mode-locked pulse train E(t) = $H_{mn}(x, y)\Sigma_q E_q \exp(-i\omega_c t) = H_{mn}(x, y)E_{pulse}(t)$  is obtained with a periodicity of  $T = 2\pi/\omega_c$ , the cavity round-trip time; here  $E_{pulse}(t)$  represents the timedependent electric-field complex amplitude. When two transverse modes (e.g., TEM<sub>00</sub> and TEM<sub>01</sub>) are simultaneously active, we have

$$E(t) = [H_{00}(x, y) + \alpha H_{01}(x, y) \exp(-i\Omega t) \\ \times \exp(i\Delta\phi)]E_{\text{pulse}}(t), \qquad (2)$$

where  $\alpha$  represent the ratio of mode amplitudes. If the phase difference  $\Delta \phi$  between modes is locked, the period of E(t) is now given by  $\omega_c/\Omega T$  because of the modulation of the transverse spatial profile. It has been shown<sup>6</sup> that the mode-locked Ti:sapphire laser is optimally operated near the edge of the cw stability region where the differential gain per unit of self-phase modulation is largest and  $\theta$  is close to  $\pi$ , which brings the resonator into a region where the phase shift  $\theta$  is extremely sensitive to the positions of the curved mirrors because it is equivalent to a confocal Fabry-Perot resonator. When  $\theta = \pi$ ,  $\omega_c/\Omega = 2$  and the period is twice the cavity round-trip time. The two regions of mirror position where PD operation is observed (Fig. 2) are consistent with this condition, given that the Kerr-lens focal length varies with intracavity power. Simple modeling of the cavity by ABCD analysis shows that the PD effect is strongly related to the position of the rod between the two mirrors and the distance between  $M_3$  and  $M_4$ . This suggests that the Kerr lens plays a significant role in the modeling, but the identification of the detailed values of particular



Fig. 2. Average output power as a function of position of  $M_3$  referred to an arbitrary origin approximately 5 cm from the end of the Ti:sapphire rod. The arrows indicate the scan direction.



Fig. 3. (a) Dashed curves, electric-field profiles of the  $\text{TEM}_{00}$  and  $\text{TEM}_{01}$  modes for a confocal resonator at times separated by the cavity round-trip time. Solid curves, total electric-field amplitude for  $\alpha = 0.8$ . (b) Propagation path of the totally mode-locked  $\text{TEM}_{00}$  and  $\text{TEM}_{01}$  pulse; the cavity is shown as an unfolded cavity with M<sub>3</sub> and M<sub>4</sub> replaced by lenses for illustration purposes; angles are exaggerated.

parameters will require more-extensive modeling incorporating a variety of nonlinear effects. The major hurdle in this modeling is the self-consistent treatment of optical nonlinearities whereby a superposition of modes changes the Kerr lens, which in turn changes the cavity, making a linear stability analysis difficult. The hysteresis behavior observed in Fig. 2 is also not unexpected for a laser operating with a nonlinear Kerr lens. However, because of the sensitivity of the Guoy phase shift to cavity parameters and laser intracavity power, reproducing all the features of Fig. 2 would require a full beam propagation simulation. Figure 3(a) illustrates how the transverse characteristics of the pulse field amplitude evolve for the case when  $\alpha = 0.8$ ,  $\theta = \pi$ , and  $\omega_c/\Omega = 2$ . A transverse sweeping action occurs, which gives rise to a single-peaked (spatially as well as temporally) pulse propagating at a slight angle to the cavity axis [Fig. 3(b)] and therefore exits the cavity at slightly different angles on successive round trips. For our laser we obtain an angle of <3 mrad by dividing the waist at the crystal face by the crystal length. This PD pulse experiences less gain than a TEM<sub>00</sub> beam because of the small waist gain volume of the laser. With a pump waist of 30  $\mu$ m, a tilt angle of 3 mrad produces a decrease in gain of 2%, consistent with the reduction of power observed experimentally. Note that if the geometry of the cavity were such that  $\omega_c/\Omega$  were equal to 3 or 4 or were incommensurate, one would observe period three, period four, or quasi-periodicity. Indeed, we have observed period triping and quadrupling effects when some of the optical elements (e.g., group-velocity dispersion prisms) were changed, misaligned, or both.

For transverse mode locking one needs a mechanism by which the  $\text{TEM}_{00}$  mode is coupled to higherorder modes to initiate the beating and then the locking. A likely mechanism is gain saturation. In our experiments the laser intracavity average power is  $\sim$ 50 kW/cm<sup>2</sup>, which is lower than but not small compared with the 300-kW/cm<sup>2</sup> saturation intensity<sup>6</sup> of Ti:sapphire. Inasmuch as the gain is not flat across the beam spatial profile, preferential saturation occurs at the center of the gain region and coupling of the TEM<sub>00</sub> to higher-order modes. The locking mechanism is believed to be the same mechanism that leads to longitudinal locking because, to have KLM operation, one expects a Gaussian profile within the gain medium for proper focusing. Hence a superposition of higher-order modes will phase lock in a way that leads to a Gaussian-like profile in the gain medium. Only if the two conditions (coupling to higher-order modes and phase locking) are simultaneously satisfied can one obtain multiple-period behavior. Otherwise the laser operates in a TEM<sub>00</sub> mode.

Finally, it is interesting to mention that, whereas transverse mode locking of Hermite-Gaussian modes will always give rise to beam sweeping, mode locking of the cylindrically symmetric Laguerre modes would give rise to a modulation of the beam waist (breathing). However, standard Ti:sapphire resonators support Hermite-Gaussian modes and not Laguerre-Gauss modes because spatial symmetry is broken owing to the Brewster faces and the astigmatism of the curved mirrors. Therefore only period doubling of the sweeping type should be observed.

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