# Enhanced second-harmonic generation in AlGaAs microring resonators

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Highly efficient second-harmonic generation can be achieved by harnessing resonance effects in microring resonator structures. We propose an angular quasi-phase-matching scheme based on the position dependence of polarization inside the ring resonator. © 2007 Optical Society of America

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Resonance effects in artificially structured materials can increase both the intensity of light and the lightmatter interaction time, and thus enhance the effects of nonlinear optical interactions. Apart from interesting effects that rely on the third-order ( $\chi^{(3)}$ ) nonlinearities (e.g., Refs. 1–4 and references therein) effects involving second-order ( $\chi^{(2)}$ ) nonlinearities have also been considered. Ilchenko *et al.*<sup>5</sup> have demonstrated enhancement of second-harmonic generation (SHG) in a periodically poled microdisk cavity. An alternative poling scheme has also been proposed, <sup>6</sup> but apparently not yet realized, in which different  $\chi^{(2)}$  materials alternate periodically with the polar angle  $\theta$  in the cavity. In this Letter we present a scheme for achieving enhanced, quasi-phase-matched SHG without requiring any artificial variation in  $\chi^{(2)}$ .

We propose using a one-channel (two-port) microring resonator structure (Fig. 1). For [100] grown AlGaAs structures, the second-order nonlinearity of the material is characterized by

$$P_i = \epsilon_0 \chi^{(2)} \sum_{j,k=1,2,3} \delta_{ijk} E_j E_k,$$

, where  $\delta_{123}=1$ ,  $\delta_{ijk}$  is symmetric under all permutations of its indices and vanishes unless (ijk) are all distinct. We consider an in-plane TE-polarized fundamental (FW) field at frequency  $\omega_F$  and an out-of-plane TM-polarized second-harmonic (SH) field at frequency  $\omega_S=2\omega_F$ . We can design a structure so that both fields are on resonance in the microring by requiring

$$k_F R = m_F, \quad k_S R = m_S, \tag{1}$$

where  $m_F$  and  $m_S$  are integers, R is the ring radius, and  $k_{F,S} = \omega_{F,S} \times n_{\text{TE,TM}}(\omega_{F,S})/c$ , where  $n_{\text{TE,TM}}$  is the effective index of the TE, TM mode. Quasi-phasematching is achieved by utilizing the dependence on  $\theta$  (see Fig. 1) of the effective nonlinear susceptibility that arises due to the variation of the local mode polarization with respect to the crystal axes; the condition that results is

$$k_S - 2k_F = s\frac{2}{R},\tag{2}$$

where  $s = \pm 1$ : note that if the first of conditions (1) and condition (2) hold, then the second of conditions (1) holds. We can satisfy these conditions, for s=1, with a device where the out-of-plane confinement would be achieved by a planar structure with three  $Al_xGa_{1-x}As$  layers (x=70%,0%,70%), and the inplane confinement realized by electron-beam lithography and dry etching of 600 nm wide, 2.9  $\mu$ m deep trench waveguides. We choose  $m_F = 100$  for the fundamental resonance of the lowest-order TE mode; then conditions (1) and (2) are satisfied with a ring radius  $R=10.299 \ \mu\text{m}$  at an operating  $\omega_F=2\pi \cdot 163.195 \text{ THz}$ (vacuum  $\lambda_o = 1.837 \ \mu$ m), and the SH is generated in a higher-order TM mode. The calculated effective indices<sup>7</sup> are  $n_{\text{TE}}(\omega_F) = 2.8388$  and  $n_{\text{TM}}(\omega_S) = 2.8672$ , with group velocities  $v_F = 78.0 \ \mu m/ps$  and  $v_S$ =55.0  $\mu$ m/ps for the FW and SH fields, respectively.

In the ring the fields satisfy the coupled mode equations



Fig. 1. (Color online) Schematic of a one-channel microring resonator structure.

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$$\begin{split} &\left(\frac{1}{v_S}\frac{\partial}{\partial t} + \frac{\partial}{\partial \zeta}\right) G_S(\zeta, t) = \mu_S(\zeta, t) \\ &\equiv i \, \eta \nu(\theta) e^{-i(k_S - 2k_F)\zeta} G_F^2(\zeta, t), \end{split}$$

$$\left(\frac{1}{v_F}\frac{\partial}{\partial t} + \frac{\partial}{\partial \zeta}\right) G_F(\zeta, t) = \mu_F(\zeta, t) \equiv i \,\eta^* \nu(\theta) e^{i(k_S - 2k_F)\zeta} \\ \times G_S(\zeta, t) G_F^*(\zeta, t),$$
(3)

where  $\zeta = R\theta$  and  $G_{F,S}(\zeta,t)$  is normalized such that  $|G_{F,S}(\zeta,t)|^2$  gives the power in the FW, SH mode;  $\nu(\theta) = \sin \theta \cos \theta$  arises because of the variation with  $\theta$  of the crystal axes determining the nonlinearity with respect to the local polarization of the waveguide modes. The coupling parameter  $\eta = e^{i\phi}/\sqrt{PA}$ , where  $\mathcal{P} \equiv 4\epsilon_0 \bar{n}^6 v_F^2 v_S / [(\chi^{(2)})^2 \omega_F \omega_S]$  has units of power; a reference index  $\bar{n}$  has been introduced for convenience in the discussion below, and the phase  $\phi$  is chosen so that  $\sqrt{A}$  is real,

$$\frac{e^{i\phi}}{\sqrt{A}} \equiv \epsilon_0^{3/2} \overline{n}^3 \int d\vec{\rho} \,\delta_{ijk} e^i_{TE}(\vec{\rho}) e^j_{TE}(\vec{\rho}) (e^k_{TM}(\vec{\rho}))^*,$$

where the two-dimensional vector  $\vec{\rho}$  ranges perpendicular to the direction of propagation, and  $\vec{e}_{\text{TE,TM}}(\vec{\rho})$  is the eigenfunction of the TE (FW), TM (SH) mode and is normalized according to

$$\epsilon_0 \int \mathrm{d}\vec{\rho} n^2(\omega_{F,S},\vec{\rho}) \mathrm{e}_{TE,TM}^*(\vec{\rho}) \cdot \mathrm{e}_{TE,TM}(\vec{\rho}) = 1;$$

A plays the role of a coupling area between the modes. Taking  $\chi^{(2)} \approx 100 \text{ pm/V}$ ,<sup>8</sup> we find  $\mathcal{P} = 0.87 \text{ MW}$  and  $A = 0.71 \ \mu \text{m}^2$  for  $\bar{n} = 3.4$ . In the channel, where resonant and phase-matching effects are not present, we neglect the nonlinearity and take Eq. (3) with  $\eta = 0$ .

Coupling between the channel and ring is taken into account in the usual way,<sup>9</sup>

$$\begin{pmatrix} G_{j2} \\ G_{j3} \end{pmatrix} = \begin{pmatrix} \sigma_j & i\kappa_j \\ i\kappa_j & \sigma_j \end{pmatrix} \begin{pmatrix} G_{j1} \\ G_{j4} \exp(ik_jL) \end{pmatrix},$$
(4)

where j specifies F or S, the positions of the fields (1)–(4) are indicated in Fig. 1, and  $L=2\pi R$  is the circumference of the ring. For simplicity we take the coupling constants  $\sigma_j$ ,  $\kappa_j$  to be real; energy conservation then requires that  $\sigma_i^2 + \kappa_i^2 = 1$  for each j. For our calculations below we take  $\kappa_{F,S}=0.199$ , although in general the constants will be different; our results do not depend in an essential way on them being the same. We note that, although typically  $\kappa_S$  for the *lowest*-order TM (SH) mode is much smaller than  $\kappa_F$ for the lowest-order TE (FW) mode, the higher-order TM (SH) mode used in our design can give  $\kappa_S$  equal to or even larger than  $\kappa_F$  for the lowest-order TE (FW) mode, depending on the coupling gap between the ring and the straight waveguide. The Q factors, defined as  $Q_{F,S} = \pi m_{F,S} / (1 - \exp(-\pi a_{F,S}R)\sigma_{F,S})$ , where  $a_{F,S}$  is the power loss coefficient in the ring for the FW, SH mode, are  $Q_F = 1.6 \times 10^4$  and  $Q_S = 3.2 \times 10^4$  for a lossless structure and  $Q_F = 8.0 \times 10^3$  and  $Q_S = 1.6 \times 10^4$  for a structure with a power loss of 26 dB/cm in the ring.

We first neglect any loss. Insight into the physics of SHG enhancement can be gleaned from looking at Eqs. (3) and (4) in the CW limit. With  $\partial/\partial t = 0$ , and under the assumption that  $G_{S1}=0$  (no incident light at  $\omega_S$ ), we find

$$G_{S2} = \alpha_S \int_3^4 \mu_S(\zeta) d\zeta,$$
  

$$G_{F3} = \alpha_F G_{F1} + \frac{\sigma_F \exp(ik_F L)}{1 - \sigma_F \exp(ik_F L)} \int_3^4 \mu_F(\zeta) d\zeta, \quad (5)$$

where the  $\alpha_j = i\kappa_j/(1-\sigma_j \exp(ik_jL))$  are enhancement factors; at a resonance  $(\exp(ik_jL)=1)$ , for our device, we have  $|\alpha_j|^2 \approx 100$ . We first consider the undepleted pump approximation, in which we can neglect  $\mu_F(\zeta)$ ;  $G_F(\zeta)$  is then independent of  $\zeta$  in the ring, and from the second of Eqs. (5) is  $G_{F3} = \alpha_F G_{F1}$ . Writing  $\nu(\theta) =$  $-i(\exp(2i\zeta/R) - \exp(-2i\zeta/R))/4$  and keeping the term that contributes near quasi-phase-matching (Eq. (2) with s=1), we find

$$\int_{3}^{4} \mu_{S}(\zeta) \mathrm{d}\zeta \simeq \frac{\eta}{4} e^{-iqL/2} L \Pi(qL) G_{F3}^{2}, \tag{6}$$

where  $q = k_S - 2k_F - 2/R$  and  $\Pi(2x) = \sin(x)/(x)$ . Writing the input fundamental power  $|G_{F1}|^2 \equiv \mathcal{P}_F$  and the output SH power  $|G_{S2}|^2 \equiv \mathcal{P}_S$ , Eqs. (5) then leads to the  $\mathcal{P}_S = f \mathcal{P}_F^2 L^2 \mathcal{P}^{-1} A^{-1},$ result that where f  $= |\alpha_S|^2 |\alpha_F|^4 \Pi^2 (qL)/16$ . Recall that in a simple onedimensional analysis of SHG, using the undepleted pump approximation and assuming the  $\chi^{(2)}$  can be fully accessed, propagation over a length L in a nondispersive medium (where phase matching automatically follows) leads to the generation of SH power through an area A given by  $\mathcal{P}_S$  but with f=1, if the parameters in  $\mathcal{P}$  are set appropriately<sup>10</sup>; hence our f plays the role of an enhancement factor. For our device we find a value  $f = 6 \times 10^4$ .

The enhancement is so large that complete conversion is possible at reasonable input powers. To estimate the condition for this, note first that from Eq. (4) we find, taking  $G_{F2}=0$ , that  $G_{F3}=iG_{F1}/\kappa_F$ . Assuming quasi-phase-matching and neglecting the variation in  $G_F(\zeta)$  over the ring, from Eq. (6) we derive an equation similar to  $\mathcal{P}_S$ , but with  $f=|\alpha_S|^2\kappa_F^{-4}/16$ ; we use this and the fact that, at complete conversion, we must have  $\mathcal{P}_S=\mathcal{P}_F$ , to estimate the value  $\bar{\mathcal{P}}_F = 16\mathcal{P}\kappa_F^4A|\alpha_S|^{-2}L^{-2}$  of the input power at which complete conversion can be achieved. For our device we find  $\bar{\mathcal{P}}_F = 38$  mW.

To confirm the validity of these approximations and demonstrate that our results hold qualitatively even for pulse propagation, we present numerical simulations<sup>11</sup> of Eqs. (3) and (4). We first consider "turning on" a CW incident field with  $\mathcal{P}_F$ =38 mW, using a half-Gaussian rising edge with a FWHM =150 ps, as shown in Fig. 2(a). It is clear that, once



Fig. 2. Time evolution of the output fields with incidence of (a) a CW fundamental wave with a half-Gaussian (FWHM=150 ps) rising edge, (b) a 150 ps Gaussian pulse, (c) same as (b) but with a power loss of 26 dB/cm (i.e., 6/cm) inside the ring.

the fields are established, essentially all the incident fundamental is indeed converted to SH. Our CW analysis above assumes a single input frequency and predicts complete conversion only for a particular input power. Nonetheless, if the frequency spectrum of the pulse is sufficiently narrow, highly efficient conversion is still possible. In Fig. 2(b) we show a simulation of a 150 ps incident Gaussian pulse. Defining a "conversion efficiency" as the ratio of output SH energy to the total output energy, we find that the conversion efficiency is 92%. The additional 40 ps delay in the output SH indicates the long cavity lifetime.

In the analysis so far, we have idealized the situation by neglecting any loss. For our device the primary loss will be due to scattering from fabrication imperfections. In the simulation shown in Fig. 2(c)we introduce a power loss of 26 dB/cm at both  $\omega_F$  and  $\omega_{\rm S}$  inside the ring in our simulations.<sup>12</sup> Since the SHG enhancement involves a long time spent by the light in the ring, the effect of loss on the total amount of SH produced is significant. Nonetheless, in the output the SH field still dominates, and the conversion efficiency as defined above is here 81%. Even if we take into account the fact that the loss at  $\omega_S$  can be larger than that at  $\omega_F$ , e.g., a power loss of 52 dB/cm at  $\omega_{\rm S}$ , we still get a conversion efficiency of 70% in our simulation, although the total amount of output is less than that in Fig. 2(c).

In realistic devices, there are also fabrication errors in the ring radius R and the effective indices

 $n_{S,F}$ . In an experiment, two independent conditions [the first of Eqs. (1) and (2)] must be satisfied despite these errors. We can do this by adjusting the frequency of the input field, and the temperature using a thermal controller. For our design, using standard parameters, we estimate a temperature adjustment of  $\pm 10$  °C would compensate for a  $10^{-3}$  relative error in R, and the temperature must be held within  $\pm 0.1$  °C to obtain a conversion efficiency larger than 50%. This should be experimentally feasible.

In summary, it is possible to use the periodic spatial variations of the fundamental TE mode polarization inside an AlGaAs microring resonator to obtain the quasi-phase-matching required for SHG processes. The resonances in the ring can dramatically enhance the SHG. More general designs can lead to the enhancement of SHG and other nonlinear processes over a range of frequencies; we plan to turn to these in future communications.

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