Makeup problems

10. Estimate the free spectral range, finesse and mode waist size of an optical cavity that can be used with the D2 transition (λ=780 nm) of rubidium atoms to perform cavity QED experiments in the strong coupling regime.

11. An atom is put into the state $\alpha|g\rangle + \beta|e\rangle$. The atom has a spontaneous decay rate γ. Calculate the expectation value of the pseudo-spin vector $\left\langle \vec{S} \right\rangle$ as a function of time, and plot the purity of the state (defined here as $r = || \left\langle \vec{S} \right\rangle || / S$) as a function of time.

12. Two oscillators are coupled by a 50:50 beamsplitter. Each oscillator starts off in the number state $|n = 1\rangle$. Write down the reduced density matrices of the oscillators after the beamsplitter interaction, and sketch/plot the Q-functions of the oscillators.

13. A calcium ion is placed in an optical cavity, close to resonance with the ${}^2S_{1/2} \rightarrow {}^2D_{5/2}$ optical transition ($\lambda \approx$ 729 nm). Determine the multipolarity of the transition, and explain how the Rabi frequency changes as a function of the ion's position in the cavity.

14. A hydrogen atom is in the $1s, F = 1, m = 0$ hyperfine state, which lies at an energy $\omega_0 = 2\pi\times$ 1.4 GHz above the $1s, F = 0, m = 0$ state. The atom is placed in a highly conducting metal box that is 3 cm on a side. Calculate the rate of decay to the $1s, F = 0, m = 0$ state.

15. A ytterbium ion in the $\left|{}^2F_{7/2}, m_J = 7/2\right\rangle$ excited state undergoes a transition to the $\left|{}^2S_{1/2}, m_J = 1/2\right\rangle$ ground state, emitting a photon at 467 nm into free space.

    (a) Identify the leading order multipole that causes this transition.

    (b) How does the spontaneous emission rate scale with the wavelength for such transitions?

16. An ensemble of N atoms in their ground state is homogeneously coupled to an electromagnetic field. The atoms have a resonance frequency ω_0, and the field has a frequency ω. The interaction with the field is turned on for a duration $T = \pi/\Omega_R$, where Ω_R is the Rabi frequency. The excited state population, N_e, is measured at the end of the experiment.

    Sketch the dependence of ΔN_e as a function of the field's frequency ω. You may assume the rotating wave approximation. Provide a qualitative explanation of why ΔN_e has this dependence.

17. Explain how squeezed states of light can be used to obtain higher phase sensitivity in a Mach-Zehnder interferometer, compared to coherent states. Limit your explanation to 1 page.

18. Prove the following identity for $\vec{\sigma}$ matrices: $f(\vec{v}\cdot\vec{\sigma}) = \mathbb{I}\left[\frac{f(v)+f(-v)}{2}\right] + \frac{\vec{v}\cdot\vec{\sigma}}{v}\left[\frac{f(v)-f(-v)}{2}\right]$

19. An atom has three states: $|g\rangle, |e\rangle, |f\rangle$, with energies $0, \omega_e, \omega_f$ respectively. An optical field in a coherent state $|\alpha \gg 1\rangle$ is used to perform spectroscopy of the $g \rightarrow e$ transition. The field can be assumed to be close to resonance for the $g \rightarrow e$ transition, and far off resonance for the $g \rightarrow f, e \rightarrow f$ transitions. The dipole matrix has the form $D = \begin{pmatrix} 0 & D_{ge} & 0 \\ D_{ge} & 0 & D_{ef} \\ 0 & D_{ef} & 0 \end{pmatrix}$.

    By how much is the measured $g \rightarrow e$ transition shifted in a Rabi spectroscopy experiment (with duration T), compared to the true value ω_e?