

Pseudo-Landau levels of Bogoliubov quasiparticles in strained nodal superconductors

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Motivated by theory and experiments on strain-induced pseudo-Landau levels (LLs) of Dirac fermions in graphene and topological materials, we consider its extension for Bogoliubov quasiparticles (QPs) in a nodal superconductor (SC). We show, using an effective low-energy description and numerical lattice calculations for a d -wave SC, that a spatial variation of the electronic hopping amplitude or a spatially varying s -wave pairing component can act as a pseudomagnetic field for the Bogoliubov QPs, leading to the formation of pseudo-LLs. We propose realizations of this phenomenon in the cuprate SCs, via strain engineering in films or nanowires, or s -wave proximity coupling in the vicinity of a nematic instability, and discuss its signatures in tunneling experiments.

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I. INTRODUCTION

The ability to tune electronic properties with strain in a wide range of quantum materials has led to the emerging area of “straintronics” [1]. Strain has been shown to be an important knob in graphene, topological materials, and oxide electronics, allowing one to tune band dispersion and topology [2–10], and to control magnetism [11,12] and ferroelectricity [13] in thin films. Uniaxial strain has also been used to shed light on fundamental questions in correlated materials, from searching for chiral $p_x \pm ip_y$ pairing in Sr_2RuO_4 [14], to understanding nematicity in pnictide superconductors [15] and in the “hidden order” state of URu_2Si_2 [16].

In graphene, a two-dimensional (2D) electronic membrane [17], strain modifies the wave-function overlap between neighboring orbitals and causes a momentum-space displacement of the massless Dirac point in the dispersion, thus simulating the effect of a vector potential [2,4,18]. A spatial variation of the strain in graphene nanobubbles and “artificial graphene” leads to colossal pseudomagnetic fields of up to ~ 300 T, and a pseudo-Landau level (pseudo-LL) spectrum [3,5,6]. Strain also induces a deformation potential which acts as a “scalar gauge potential”; the corresponding in-plane electric fields can lead to a breakdown of the pseudo-LLs [18–21]. There have been theoretical studies of Josephson coupling through pseudo-LLs [22,23], and interaction effects which can lead to exotic correlated states [24,25]. Strain effects have also been generalized to three-dimensional (3D) Dirac and Weyl semimetals [26–29], Kitaev spin liquids [30], and atoms in optical lattices [31,32].

In light of these developments, we address in this paper the important question of how these phenomena manifest themselves in superconducting phases of matter. Specifically, we consider the possibility of engineering time-reversal invariant pseudo-gauge fields for Bogoliubov quasiparticle (QP) excitations of nodal superconductors (SCs). Our key observation is that the QP Dirac nodes of the SC will shift in momentum space under the modification of the single-particle dispersion or the form of the pairing gap. Thus, spatial variations of the dispersion or the pairing term can mimic a spatially varying gauge field. Using an effective low-energy theory for 2D d -wave SCs as well as a numerical lattice model study, we show that this induces pseudo-LLs

of Bogoliubov QPs and discuss its signatures in the spatially resolved tunneling density of states (TDOS).

Our work highlights two key differences between strained nodal SCs and materials such as graphene or Dirac-Weyl semimetals. (i) Unlike electrons, Bogoliubov QPs do not have a well-defined electrical charge and do not couple directly to external orbital magnetic fields. Thus, strain engineering provides a unique window to explore LL physics of Bogoliubov QPs. (ii) We show that strain variations in a d -wave SC with time-reversal symmetry cannot induce a pseudo-“scalar potential” for Bogoliubov QPs. This is unlike the impact of the deformation potential for graphene. In this regard, pseudo-LLs of Bogoliubov QPs are more robust and are “symmetry protected.”

We suggest two routes to realizing this physics in the cuprate SCs: via strain engineering in thin films and nanowires, or via edge effects or s -wave proximity coupling in the vicinity of an isotropic to nematic SC quantum phase transition (QPT) [33]. Our study sheds light on how inhomogeneous strain can reorganize the low-energy spectrum of nodal SCs.

II. EFFECTIVE LOW-ENERGY THEORY

The low-energy excitations of a uniform 2D d -wave SC on a square lattice reside near the two pairs of gap nodes $\mathbf{K}_{\pm 1} \equiv \pm(K, K)$ and $\mathbf{K}_{\pm 2} \equiv \pm(K, -K)$ as in Fig. 1(a). We combine the slowly varying fermion fields near the node pairs into Nambu spinors $\Psi_{\ell\alpha}^\dagger(\mathbf{r}) \equiv (\psi_{\ell\alpha}^\dagger(\mathbf{r}), \epsilon_{\alpha\nu} \psi_{-\ell\nu}(\mathbf{r}))$, where α, ν are spin labels (\uparrow or \downarrow), and $\ell = 1, 2$ labels the nodes $\mathbf{K}_{1,2}$. The low-energy excitations of a nodal SC are described by the effective Dirac Hamiltonian $H_0 = \sum_{\ell,\alpha} \int d^2\mathbf{r} \Psi_{\ell\alpha}^\dagger(\mathbf{r}) \mathfrak{h}_0^{(\ell)} \Psi_{\ell\alpha}(\mathbf{r})$, with

$$\mathfrak{h}_0^{(\ell)} = -i\sigma^z \vec{v}_f^{(\ell)} \cdot \vec{\nabla} - i\sigma^x \vec{v}_\Delta^{(\ell)} \cdot \vec{\nabla}, \quad (1)$$

where $\vec{v}_f^{(\ell)}, \vec{v}_\Delta^{(\ell)}$ denote the Fermi velocity and the gap velocity (respectively, normal and tangential to the Fermi surface), and $\sigma^{x,z}$ are Pauli matrices. Diagonalizing H_0 in momentum space leads to the massless Dirac dispersion $E_\ell(\mathbf{k}) = (v_f^2 k_x^2 + v_\Delta^2 k_y^2)^{1/2}$ where (k_x, k_y) denotes the deviation in momentum from \mathbf{K}_ℓ [with local coordinate axes as shown in Fig. 1(a)], and the Dirac cone anisotropy is set by v_f/v_Δ .

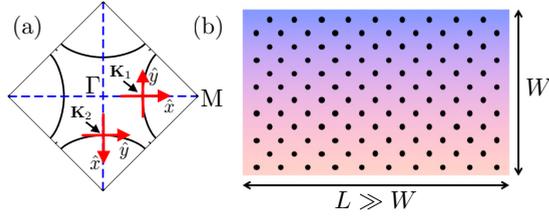


FIG. 1. (a) Rotated Brillouin zone for the square lattice showing schematic Fermi surface (solid, black) for optimal hole-doped cuprate SCs. Quasiparticle Dirac nodes are located at $\pm\mathbf{K}_1$ and $\pm\mathbf{K}_2$, and we show local coordinate axes used in our low-energy theory. (b) Strip geometry (not to scale) used in the numerics with width W and length $L \gg W$. Shading gradient illustrates spatial variation in the pairing or hopping amplitude across the strip.

We next turn to the effect of time-reversal invariant slow spatial variations in the hopping and pairing amplitudes of this nodal SC, which adds to the microscopic lattice Hamiltonian terms of the form

$$\delta H_1 = -\frac{1}{2} \sum_{\mathbf{R}, \eta, \alpha} \delta t_\eta(\mathbf{R}) (c_{\mathbf{R}, \alpha}^\dagger c_{\mathbf{R}+\eta, \alpha} + \text{H.c.}), \quad (2)$$

$$\delta H_2 = \frac{1}{8} \sum_{\mathbf{R}, \eta} \delta \Delta_\eta(\mathbf{R}) (c_{\mathbf{R}, \uparrow}^\dagger c_{\mathbf{R}+\eta, \downarrow}^\dagger - c_{\mathbf{R}, \downarrow}^\dagger c_{\mathbf{R}+\eta, \uparrow}^\dagger + \text{H.c.}), \quad (3)$$

where η denotes the set of neighbors of site \mathbf{R} and H.c. stands for Hermitian conjugate. A low-energy expansion of the fermion fields leads to the modified Hamiltonian

$$\mathfrak{h}^{(\ell)} = \begin{pmatrix} -i v_f \partial_x + f_\ell(\mathbf{r}) & -i s_\ell v_\Delta \partial_y + g_\ell(\mathbf{r}) \\ -i s_\ell v_\Delta \partial_y + g_\ell(\mathbf{r}) & i v_f \partial_x - f_\ell(\mathbf{r}) \end{pmatrix}, \quad (4)$$

where $s_\ell = (-1)^\ell$, with

$$f_\ell(\mathbf{r}) = - \sum_{\eta} \delta t_\eta(\mathbf{r}) \cos(\mathbf{K}_\ell \cdot \eta), \quad (5)$$

$$g_\ell(\mathbf{r}) = \frac{1}{4} \sum_{\eta} \delta \Delta_\eta(\mathbf{r}) \cos(\mathbf{K}_\ell \cdot \eta), \quad (6)$$

and we have implicitly assumed that we have rotated \mathbf{r} into the local coordinate axes for node ℓ . Note that a conventional deformation potential or spatially varying chemical potential may also be included in $f_\ell(\mathbf{r})$ in Eq. 4. We can recast this Hamiltonian as

$$\mathfrak{h}^{(\ell)} = v_f \sigma^z \left[-i \partial_x + \mathcal{A}_x^{(\ell)}(\mathbf{r}) \right] + s_\ell v_\Delta \sigma^x \left[-i \partial_y + \mathcal{A}_y^{(\ell)}(\mathbf{r}) \right], \quad (7)$$

where we have defined the ‘‘vector potential’’ $\vec{\mathcal{A}}^{(\ell)}$ via $v_f \mathcal{A}_x^{(\ell)}(\mathbf{r}) \equiv f_\ell(\mathbf{r})$ and $v_\Delta \mathcal{A}_y^{(\ell)}(\mathbf{r}) \equiv s_\ell g_\ell(\mathbf{r})$. Thus, slow spatial modulations of parameters in a nodal superconductor will lead to an effective low-energy theory of Dirac quasiparticles coupled to a spatially varying vector potential.

The issue of whether additional gauge potentials (e.g., a ‘‘scalar gauge potential’’ which minimally couples to time derivatives rather than space derivatives) can arise in a strained SC amounts to asking if any other Pauli matrix components are permitted in $\mathfrak{h}^{(\ell)}$. To address this, we note that terms proportional to the identity matrix will act as a valley-odd chemical potential, while a component proportional to σ^y will correspond to complex pairing. Both terms are forbidden by

time-reversal and spin-rotation symmetries in a d -wave SC, and thus cannot destabilize the pseudo-LLs; in this sense, the pseudo-LLs may be regarded as ‘‘symmetry protected’’ (see Appendix A for details). The key point is that slow modulations of the parameters of a nodal superconductor will leave the nodal quasiparticle excitations pinned to zero energy but can displace it in momentum space. Thus, d -wave Bogoliubov QPs, unlike electrons in graphene, do not experience an inhomogeneous ‘‘scalar’’ gauge potential [18,21]. However, breaking time-reversal symmetry, for instance with a supercurrent, will lead to a Doppler shift for the QPs [34], shifting the energy of the nodal excitations, which thus provides an analog of a scalar potential.

III. PSEUDO-LANDAU LEVELS

We next turn to the spectrum of $\mathfrak{h}^{(\ell)}(\mathbf{r})$ for two illustrative cases, with $\vec{\mathcal{A}}$ induced by variations in the pairing gap or hopping amplitude, to show the emergence of pseudo-LLs. We then supplement the continuum theory with numerical results on a lattice realization.

A. Pseudo-LLs from gap variations

Let us impose an additional extended s -wave pairing with a uniform gradient along the $[1,1]$ direction, which translates to $\delta \Delta_{+x}(\mathbf{r}) = \delta \Delta_{+y}(\mathbf{r}) = (x_a/a_0 + x_b/a_0 + 1/2)\Delta_s$. Here, (x_a, x_b) refer to (global) coordinates corresponding to the a and b crystal axes, and a_0 is the lattice constant. Using this, we find $f_\ell(\mathbf{r}) = 0$, while, in the local coordinates at $\ell = 1, 2$, we have $g_1(\mathbf{r}) = \beta v_\Delta x$ and $g_2(\mathbf{r}) = \beta v_\Delta y$, with $\beta \equiv \sqrt{2} \frac{\Delta_s}{v_\Delta a_0} \cos K$.

For node pair $\ell = 2$, this leads to $\vec{\mathcal{A}}^{(2)} = (0, \beta y)$, which yields $\vec{\mathcal{B}}^{(2)} = 0$. In this case, the energy spectrum is unaffected by the modulation, while the wave functions are obtained by a gauge rotation as $e^{-\frac{i}{2}\beta y^2} \Psi^{(2)}(\mathbf{r})$, where $\Psi^{(2)}(\mathbf{r})$ is the Nambu spinor wave function of the uniform d -wave SC for node pair $\ell = 2$.

For node pair $\ell = 1$, we arrive at $\vec{\mathcal{A}}^{(1)} = (0, -\beta x)$, i.e., the Landau gauge for a pseudomagnetic field $\vec{\mathcal{B}}^{(1)} = -\beta \hat{z}$. Setting the Nambu wave function $\Psi^{(1)}(\mathbf{r}) = e^{iky} \Phi^{(1)}(x)$, we get (see Appendix B)

$$\left[-i v_f \sigma^z \partial_x + \beta v_\Delta \sigma^x \left(x - \frac{k}{\beta} \right) \right] \Phi^{(1)}(x) = E \Phi^{(1)}(x). \quad (8)$$

Defining $|\uparrow\rangle = \frac{1}{\sqrt{2}}(1, i \operatorname{sgn} \beta)^T$ and $|\downarrow\rangle = \frac{1}{\sqrt{2}}(1, -i \operatorname{sgn} \beta)^T$, we find a zero-energy eigenstate $|\Phi_{k0}\rangle = |0\rangle_k |\downarrow\rangle$ and nonzero energy eigenstates

$$|\Phi_{kn\pm}\rangle = \frac{1}{\sqrt{2}}(|n-1\rangle_k |\uparrow\rangle \pm i |n\rangle_k |\downarrow\rangle), \quad (9)$$

where the subscript \pm denotes states with energies $\pm \sqrt{2} |\beta| v_\Delta v_f n$ (with integer $n \geq 1$). Here, $|n\rangle_k$ is the n th eigenstate of a harmonic oscillator centered at k/β , with a mean square width $\langle x^2 \rangle = (n+1/2) \frac{v_f}{|\beta| v_\Delta}$. We confirm these findings below within a lattice model of a d -wave superconducting strip.

B. Pseudo-LLs from hopping variations

Next, let us consider a uniform spatial gradient in the hopping along the $[1,1]$ direction, given by $\delta t_{+x}(\mathbf{r}) = \delta t_{+y}(\mathbf{r}) = -(x_a/a_0 + x_b/a_0 + 1/2)t_s$, where t_s sets the scale of the hopping distortion. This results in $g_\ell(\mathbf{r})=0$ and, in local coordinates, $f_1(\mathbf{r})=\beta v_f x$ and $f_2(\mathbf{r})=\beta v_f y$, where $\beta \equiv 4\sqrt{2}\frac{t_s}{v_f a_0} \cos K$. This, in turn, leads to $\vec{A}^{(1)}=(\beta x, 0)$, which corresponds to zero pseudomagnetic field, while $\vec{A}^{(2)}=(\beta y, 0)$ yields a pseudomagnetic field $\vec{B}^{(2)} = -\beta \hat{z}$, which supports pseudo-LL energies identical to the case with gap variation for the same choice of β (see Appendix C). A similar pseudovector potential can also be realized by a spatially varying nematic distortion of the second-neighbor hopping, with $\delta t_{+x+y}(\mathbf{r}) = -(x_a/a_0 + x_b/a_0 + 1)t_s$ and $\delta t_{+x-y}(\mathbf{r}) = (x_a/a_0 + x_b/a_0)t_s$, which yields $\vec{B}^{(1)}=0$ and $\vec{B}^{(2)} = -\beta \hat{z}$, with $\beta \equiv 4\sqrt{2}\frac{t_s}{v_f a_0} \sin^2 K$. We note that while these examples are “gauge equivalent” to the earlier gap variation case, their physical realizations are distinct since we are changing the hopping rather than the gap, thus directly controlling the “vector potential.”

IV. LATTICE MODEL RESULTS

To check the validity of the low-energy linearized Dirac theory, we numerically diagonalized the full lattice Bogoliubov-de Gennes (BdG) Hamiltonian using a strip geometry with $(1,1)$ edges [see Fig. 1(b)]. The strip width is W ; the transverse direction, along which periodic boundary conditions were used, has length $L \gg W$. Analogous results for the $(1,0)$ -edged strip are presented in Appendix F. We pick a nearest-neighbor hopping amplitude $t=1$, next-neighbor hopping $t'=-0.25t$, electron filling $\bar{n}=0.85$, and a d -wave gap $\Delta_d=0.25t$, such that $v_f/v_\Delta \approx 13$; these parameters are chosen so as to be representative of the hole-doped cuprate SCs.

Figure 2(a) shows the spectrum of the $(1,1)$ -edged strip as a function of the momentum k_L along the long direction L , in the absence of any imposed spatial variation for $W=500\sqrt{2}a_0$. The spectrum exhibits d -wave Dirac nodes projected onto the Brillouin zone of the strip; the velocity anisotropy $v_f/v_\Delta \gg 1$ is evident in the dispersion slopes of the outer versus inner nodes. In addition, we find zero-energy Andreev bound states (ABSs) expected for a d -wave SC in this geometry [35–38].

Figure 2(b) shows the spectrum with a nonzero gradient in the hopping amplitude across the strip width, which leads to a pseudo-LL spectrum at the outer Dirac nodes; we have chosen to plot the spectrum near the Dirac node indicated by the circle in Fig. 2(a), for strip width $W=500\sqrt{2}a_0$ and a maximum change $\delta t \sim 0.1t$ at the edge. Figure 2(c) shows the effect of an extended s -wave pairing gradient along the strip width, which leads to pseudo-LL formation at the central Dirac node. Here, we have chosen $W=2000\sqrt{2}a_0$ and a maximum s -wave gap $\Delta_s \sim 0.4\Delta_d$ at the edge. The low-energy spectra in Figs. 2(b) and H.c.(c) are in quantitative agreement with our analytical results. The spectrum for the $(1,0)$ -edged strip (see Appendix F) displays similar strain-induced pseudo-LLs; the key difference is in the absence of ABSs for the unstrained d -wave SC in this geometry.

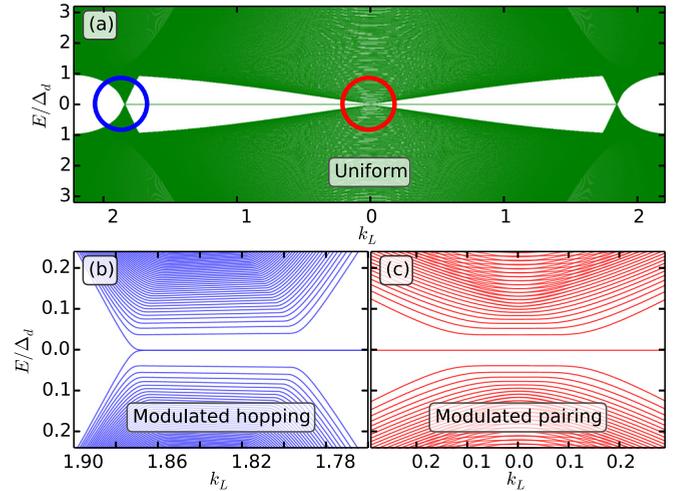


FIG. 2. (a) Spectrum of uniform d -wave SC on a $(1,1)$ -edged strip versus momentum k_L along the L direction, showing Dirac nodes and zero-energy ABSs. Circles indicate regions shown in the next two panels. (b) Formation of flat pseudo-Landau levels near the outer Dirac nodes due to uniform hopping-amplitude gradient in the $[1,1]$ direction; shown here is the near-node region indicated in (a). (c) Similar to (b) but with extended s -wave pairing gradient, which induces pseudo-LLs near the central Dirac node indicated in (a).

V. EXPERIMENTAL SIGNATURE OF PSEUDO-LLS

As in the case of strained graphene, scanning tunneling spectroscopy (STS) experiments which probe the TDOS may provide the most direct route to observing the QP pseudo-LLs. For weak pseudomagnetic fields, the peaks in density of states due to pseudo-LLs may be visible in microwave spectroscopy. Below, we first provide analytical expressions for the bulk TDOS expected within our continuum low-energy theory. We then present numerical results on the lattice model (see Fig. 3) which goes beyond the continuum theory by incorporating the effects of quantum confinement of the Bogoliubov QPs to the strip, as well as the impact of ABSs at the edges.

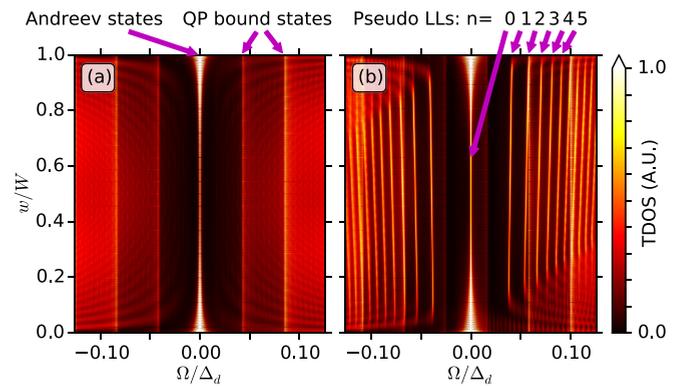


FIG. 3. Low-energy TDOS versus energy Ω/Δ_d (scaled to the d -wave gap), from diagonalization of BdG Hamiltonian in the strip geometry, plotted across scaled strip width $0 < w/W < 1$. (a) Uniform d -wave SC, showing ABSs near zero energy localized near $w/W = 0, 1$, and QP bound-state TDOS exhibiting rapid spatial oscillations. (b) Hopping gradient case showing extra pseudo-LL peaks.

In tunneling experiments, the TDOS in the continuum theory will have two contributions in the bulk. At nodes where the vector potential acts as pure gauge, it will only induce a phase shift for the fermion operators, leading to a TDOS contribution identical to a uniform d -wave SC. At nodes where the QPs sense a pseudomagnetic field, there will be discrete pseudo-LLs. These lead to a total TDOS (details in Appendix D)

$$N(\Omega) \approx \frac{|\Omega|}{\pi v_f v_\Delta} + \frac{|\beta|}{\pi} \sum_n \delta(\Omega - \lambda_n), \quad (10)$$

where $n = 0, \pm 1, \pm 2, \dots$ and $\lambda_n = \sqrt{2\beta v_f v_\Delta |n|} \text{sgn}(n)$.

We have also computed the TDOS numerically for the lattice model in the above strip geometry. Confinement to the strip then leads to QP subbands with minima at discrete energies $\sim p\pi v_\Delta/W$ and $\sim p\pi v_f/W$ for nodes $\mathbf{K}_1, \mathbf{K}_2$ respectively ($p = \text{nonzero integer}$), as well as ABSs at the strip edges. As seen from Fig. 3, the TDOS for the strip exhibits three key features. (i) Without or with a gradient in the hopping amplitude, we see the zero-energy peaks in the TDOS at the top and bottom edges reflecting the presence of ABSs; the spectral weight from these ABSs weakly leaks into the bulk. As shown in Appendix F, the ABSs and their contribution to the TDOS is absent for a (1,0)-edged strip. (ii) In the bulk (i.e., away from the edges), one set of indicated peaks exhibits rapid spatial oscillation of the TDOS across the strip width. These peaks arise when the energy Ω crosses the minimum Ω_0^s (at $k_L=0$) of each subband s in the spectrum, leading to a $\sim 1/\sqrt{\Omega - \Omega_0^s}$ divergence in the TDOS. These QP bound states (see Appendix E) arise due to internode scattering $\mathbf{K}_1 \leftrightarrow -\mathbf{K}_1$. There are additional weaker features with longer-length-scale spatial variations arising from intranode scattering at $\pm\mathbf{K}_2$. Both contributions are present even in the absence of a gradient; see Fig. 3(a). (iii) Finally, the hopping gradient induces an extra set of indicated pseudo-LL peaks seen in Fig. 3(b) where the TDOS is nearly constant across the strip. The spatial dependence of the TDOS distinguishes the pseudo-LL peaks from QP bound states.

VI. EXPERIMENTAL REALIZATIONS

A. Strained nanowires or films

One route to tuning the spatial variation of the electron hopping and pairing amplitudes discussed above is to strain a cuprate thin film or nanowire. Unlike graphene, which has a simple single-particle description of its electronic bands, it is necessary here to include electron interactions in order to study the microscopic impact of strain on the d -wave SC. The cuprates may be modeled by a tJ Hamiltonian $H_{tJ} = -g_t \sum_{i,j,\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + g_J J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$, with bare nearest- and next-neighbor hoppings t_0 and $t'_0 \approx -0.3t_0$, respectively, and nearest-neighbor spin exchange $J = 4t_0^2/U \approx 0.3t_0$. We set $t_0 = 450$ meV which leads to $J = 135$ meV. The coefficients g_t, g_J represent renormalization factors that crudely account for strong correlation effects. Motivated by slave-boson [39] and renormalized mean field theory calculations [40,41], we pick $g_t = 2p/(1+p)$ and $g_J = 1$, where p is the hole doping (see Appendix G for details). Such a mean field approach captures a variety of experimental observations on the d -wave

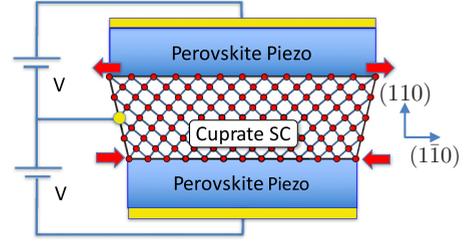


FIG. 4. Trilayer heterostructure with cuprate SC thin film epitaxially sandwiched between two piezoelectric perovskite films along the (110) surface. An inhomogeneous strain can be induced in the cuprate layer by asymmetrically polarizing the two piezolayers. Metallic outer gates (yellow regions) are used to apply the piezovoltages, with the cuprate layer serving as the common inner gate. For typical values of piezoconstant ($d_{31} \sim 50\text{--}275$ pm/V) and dielectric breakdown field (~ 25 MV/m) for piezoelectric perovskites [44,45] such as $\text{Pb}(\text{Zr}_x\text{Ti}_{1-x})\text{O}_3$, we estimate that lattice strains $\sim 0.1\text{--}1\%$ can be induced in the cuprate layer.

cuprate SCs; we therefore view it as a useful tool to estimate the pseudo-LL gap.

Here, we consider the effects of inhomogeneous strain that can be induced using a piezoelectric thin-film heterostructure schematically depicted and discussed in Fig. 4. Such piezoinduced strain will lead to a gradient in the hopping $\delta t_0(\mathbf{r})$ as well as a change in the superexchange interaction $\delta J(\mathbf{r}) \approx (8t_0/U)\delta t_0(\mathbf{r})$ across the strip. This induces a gradient in the effective hopping and pairing amplitude in the BdG equation. Raman scattering studies of La_2CuO_4 under hydrostatic pressure [42] indicate that a $\mp 0.5\%$ change in the lattice constant leads to $\delta J/J \approx \pm 5\%$, indirectly implying a change in the bare hopping amplitude $\delta t_0/t_0 \approx \pm 2.5\%$ in the underlying tJ model. A self-consistent solution to the mean field equations in the SC state at a hole doping $p = 0.15$ shows that such a uniform change leads to a $\approx \pm 7\%$ change in the d -wave pairing gap and $\approx \pm 3\%$ change in the renormalized hopping. A gradient in the d -wave gap does not induce any pseudo-LLs; however, the hopping gradient can in fact induce pseudo-LLs as discussed above. For a (110)-edged film of thickness $\sim 700a_0$, or a nanowire of similar width (≈ 270 nm) which is experimentally realizable [43] and similar to the strip geometry explored here, we estimate that a hopping gradient with a realistic 0.5%–1% maximal strain across the sample will generate a first-excited pseudo-LL at $E_1 \sim 1$ meV; this can be probed by c -axis tunneling. A fully self-consistent inhomogeneous BdG study of this physics is challenging due to the large system sizes involved; we defer this to future work.

B. Proximity to nematic order

A different route to realizing pseudo-LLs is to note that the onset of nematic order in a tetragonal d -wave SC spontaneously breaks the C_4 point-group symmetry and will induce an extended s -wave component to the pair field [33]. There is evidence that the cuprates are proximate to such a QPT [46–51], so that an edge-induced s -wave pairing component will exhibit slow spatial decay, leading naturally to a gap variation needed to form pseudo-LLs. Tuning near such a critical point, or using proximity effect coupling to an s -wave

SC, can tune the decay length and amplitude of the s -wave gap, thus controlling the pseudomagnetic field and permitting further experimental tests.

VII. SUMMARY

We have proposed inhomogeneously strained nodal SCs as systems to realize pseudo-gauge fields and pseudo-LLs for Bogoliubov QPs, and suggested experimental routes and signatures to observe such physics in candidate materials such as the cuprate d -wave SCs. We note that even accidental SC Dirac nodes will show similar physics. Further research directions include understanding the impact of such inhomogeneous strains on the superconducting transition temperature, its interplay with real magnetic fields and vortices, and extensions to materials like CeCoIn₅, iron pnictides, and candidate topological SCs like Sr₂RuO₄.

Note added. Recently, a closely related work appeared by Nica and Franz [52]. Our results, where they overlap, are in agreement.

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APPENDIX A: ABSENCE OF “SCALAR GAUGE POTENTIAL” IN BdG EQUATION

Inhomogeneous strain effects also lead to a deformation potential, which in graphene produces a *scalar* gauge potential in addition to the pseudovector potential [2,4,18–21]. Here, we argue that no such scalar potential, which may significantly alter the low-energy LL structure, or even cause its collapse [21], can arise in time-reversal symmetric spin-singlet superconducting systems, such as the one we consider.

The key physical idea is that the BdG Hamiltonian for a singlet SC with time-reversal symmetry only permits two of the four Pauli matrices: the corresponding coefficients are in fact the two components of the vector potential identified in the main body of this paper. Thus, any analog of the “scalar deformation potential” here will necessarily break time-reversal symmetry or lead to singlet-triplet mixing. Such terms will be allowed in a more general setting, for example, if spin-orbit coupling is present and inversion symmetry or time-reversal symmetry is broken, but not in the cases studied here.

The Pauli matrix components that can enter the Hamiltonian of Eq. (4) of the paper are constrained by symmetry. This is most easily seen by considering the BdG Hamiltonian in real space,

$$H_{\text{BdG}} = \sum_{i,j} \psi_i^\dagger h_{ij} \psi_j, \quad (\text{A1})$$

$$h_{ij} = \begin{pmatrix} d_{ij}^0 + d_{ij}^3 & \Delta_{ij} \\ \Delta_{ji}^* & d_{ij}^0 - d_{ij}^3 \end{pmatrix}, \quad (\text{A2})$$

where $\psi_i^\dagger = (c_{i\uparrow}^\dagger, c_{i\downarrow}^\dagger)$ is the Nambu spinor at site i , and $d_{ij}^0, d_{ij}^3, \Delta_{ij}$ are complex numbers, with Hermiticity imposing the constraint that $d_{ij}^0 = (d_{ji}^0)^*$ and $d_{ij}^3 = (d_{ji}^3)^*$:

(a) Time-reversal symmetry, which sends $c_{i\uparrow} \rightarrow c_{i\downarrow}$, $c_{i\downarrow} \rightarrow -c_{i\uparrow}$, and complex-conjugates all complex numbers, leads to the additional restrictions (i) $d_{ij}^0 = 0$ and (ii) $\Delta_{ij} = \Delta_{ji}^*$.

(b) Spin-rotation symmetry and singlet pairing further impose the constraints $d_{ij}^3 = (d_{ij}^3)^*$ and $\Delta_{ij} = \Delta_{ij}^*$.

With these ingredients, the Hamiltonian matrix $h_{ij} = d_{ij}^3 \sigma^3 + \Delta_{ij} \sigma^1$, where d_{ij}^3 and Δ_{ij} are real numbers. Thus, time-reversal symmetry and spin-rotation symmetry, respectively, require that the coefficients of σ^0 (which corresponds to a valley-odd chemical potential) and σ^2 (which corresponds to a complex pairing component) both vanish.

Such a Hamiltonian captures a BdG SC with arbitrary spatial modulations in hopping and pairing amplitudes, and an appropriate low-energy “Dirac node” expansion recovers Eq. (4) of our paper, and only permits the two components of the vector potential which we have shown leads to the formation of pseudo-LLs. Any additional scalar potential is thus symmetry forbidden. Breaking such symmetries, for instance with a supercurrent that breaks time-reversal symmetry, leads to a Doppler shift for the QPs, which is an analog of a scalar potential.

APPENDIX B: DIRAC BdG SOLUTION: GAP VARIATIONS

Start with the Hamiltonian at node $\ell = 1$ for the case discussed in the main text where pseudo-LLs arise from gap variations:

$$H = \left[-i v_f \sigma^z \partial_x + \beta v_\Delta \sigma^x \left(x - \frac{k}{\beta} \right) \right]. \quad (\text{B1})$$

Note that $(\text{sgn}\beta\sigma^y)$ anticommutes with this Hamiltonian, so that if $|\Phi\rangle$ is an eigenstate of H with energy E , then $(\text{sgn}\beta\sigma^y)|\Phi\rangle$ is a solution with energy $-E$. (Here, $\text{sgn}\beta = \beta/|\beta|$.) This is the BdG particle-hole symmetry. Let us define

$$-i\partial_x = i\sqrt{\frac{|\beta|v_\Delta}{2v_f}}(a^\dagger - a), \quad (\text{B2})$$

$$\left(x - \frac{k}{\beta} \right) = \sqrt{\frac{v_f}{2|\beta|v_\Delta}}(a^\dagger + a), \quad (\text{B3})$$

so we get

$$H = \sqrt{2|\beta|v_f v_\Delta} \left[a^\dagger \frac{(\sigma^x \text{sgn}\beta + i\sigma^z)}{2} + a \frac{(\sigma^x \text{sgn}\beta - i\sigma^z)}{2} \right]. \quad (\text{B4})$$

Define spinors

$$|\uparrow\rangle \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \text{sgn}\beta \end{pmatrix}; \quad |\downarrow\rangle \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \text{sgn}\beta \end{pmatrix}. \quad (\text{B5})$$

Then, the Hamiltonian is of the Jaynes-Cummings type

$$H = \sqrt{2|\beta|v_f v_\Delta} [i a^\dagger S^- - i a S^+], \quad (\text{B6})$$

where S^\pm act as raising/lowering operators on the above spin- $\frac{1}{2}$ states. Let $|n\rangle$ denote harmonic oscillator states (with $n \geq 0$)

centered at k/β which are generated by a, a^\dagger . Then, we have a zero-energy eigenstate

$$|\Phi_0\rangle = |0\rangle |\downarrow\rangle \quad (\text{B7})$$

and nonzero energy solutions

$$|\Phi_{n\pm}\rangle = \frac{|n-1\rangle |\uparrow\rangle \pm i |n\rangle |\downarrow\rangle}{\sqrt{2}} \quad (\text{B8})$$

with respective energies $\pm\sqrt{2|\beta|v_f v_\Delta n}$. More explicitly, the wave functions are given by

$$\Phi_{k0}(x) = \frac{1}{\sqrt{2}} \varphi_0\left(x - \frac{k}{\beta}\right) \begin{pmatrix} 1 \\ -i \operatorname{sgn}\beta \end{pmatrix}, \quad (\text{B9})$$

$$\Phi_{kn\pm}(x) = \frac{1}{2} \begin{pmatrix} \varphi_{n-1}\left(x - \frac{k}{\beta}\right) \pm i \varphi_n\left(x - \frac{k}{\beta}\right) \\ \operatorname{sgn}\beta [i \varphi_{n-1}\left(x - \frac{k}{\beta}\right) \pm \varphi_n\left(x - \frac{k}{\beta}\right)] \end{pmatrix}, \quad (\text{B10})$$

where $\varphi_n(x)$ is the n th harmonic oscillator ground state. We can then define quasiparticle operators γ for the node pair $\ell = \pm 1$, so that

$$\begin{aligned} \Psi_{1\alpha}(\mathbf{r}) &= \begin{pmatrix} \psi_{1,\alpha}(\mathbf{r}) \\ \epsilon_{\alpha\nu} \psi_{-1,\nu}^\dagger(\mathbf{r}) \end{pmatrix} = \frac{1}{\sqrt{L}} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{y}} \left[\gamma_{0,\alpha}(\mathbf{k}) \Phi_{k0}(x) \right. \\ &\quad \left. + \sum_{n>0} (\Phi_{kn+}(x) \quad \Phi_{kn-}(x)) \begin{pmatrix} \gamma_{n,1,\alpha}(\mathbf{k}) \\ \epsilon_{\alpha\nu} \gamma_{n,-1,\nu}^\dagger(-\mathbf{k}) \end{pmatrix} \right]. \end{aligned} \quad (\text{B11})$$

In terms of these, the Hamiltonian is given by

$$H = \sum_{k,\alpha,n>0} \sqrt{2|\beta|v_f v_\Delta n} (\gamma_{n1\alpha}^\dagger(k) \gamma_{n1\alpha}(k) + \gamma_{n2\alpha}^\dagger(k) \gamma_{n2\alpha}(k)). \quad (\text{B12})$$

APPENDIX C: DIRAC BdG SOLUTION: HOPPING VARIATIONS

Start with the Hamiltonian at node $\ell = 2$ for the case discussed in the main text where pseudo-LLs arise from hopping variations. Assume plane waves along the x direction. Then,

$$H = \beta v_f \sigma^z \left(y + \frac{k}{\beta}\right) - i v_\Delta \sigma^x \partial_y. \quad (\text{C1})$$

Let us define

$$-i \partial_y = i \sqrt{\frac{|\beta|v_f}{2v_\Delta}} (a^\dagger - a), \quad (\text{C2})$$

$$\left(y + \frac{k}{\beta}\right) = \sqrt{\frac{v_\Delta}{2|\beta|v_f}} (a^\dagger + a), \quad (\text{C3})$$

so we get

$$H = \sqrt{2|\beta|v_f v_\Delta} \left[a^\dagger \frac{(\sigma^z \operatorname{sgn}\beta + i \sigma^x)}{2} + a \frac{(\sigma^z \operatorname{sgn}\beta - i \sigma^x)}{2} \right]. \quad (\text{C4})$$

Define spinors

$$|\uparrow\rangle \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \operatorname{sgn}\beta \end{pmatrix}; \quad |\downarrow\rangle \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \operatorname{sgn}\beta \end{pmatrix}. \quad (\text{C5})$$

Then, the Hamiltonian is of the Jaynes-Cummings type

$$H = \sqrt{2|\beta|v_f v_\Delta} [i a^\dagger S^- - i a S^+], \quad (\text{C6})$$

where S^\pm act as raising/lowering operators on the above spin- $\frac{1}{2}$ states. Let $|n\rangle$ denote harmonic oscillator states (with $n \geq 0$) centered at $y = -k/\beta$ which are generated by a, a^\dagger . Then, we have a zero-energy eigenstate

$$|\Phi_0\rangle = |0\rangle |\downarrow\rangle \quad (\text{C7})$$

and nonzero energy solutions

$$|\Phi_{n\pm}\rangle = \frac{|n-1\rangle |\uparrow\rangle \pm i |n\rangle |\downarrow\rangle}{\sqrt{2}} \quad (\text{C8})$$

with respective energies $\pm\sqrt{2|\beta|v_f v_\Delta n}$. More explicitly, the wave functions are given by

$$\Phi_{k0}(y) = \frac{1}{\sqrt{2}} \varphi_0\left(y + \frac{k}{\beta}\right) \begin{pmatrix} 1 \\ i \operatorname{sgn}\beta \end{pmatrix}, \quad (\text{C9})$$

$$\Phi_{kn\pm}(y) = \frac{1}{2} \begin{pmatrix} \varphi_{n-1}\left(y + \frac{k}{\beta}\right) \pm i \varphi_n\left(y + \frac{k}{\beta}\right) \\ -\operatorname{sgn}\beta [i \varphi_{n-1}\left(y + \frac{k}{\beta}\right) \pm \varphi_n\left(y + \frac{k}{\beta}\right)] \end{pmatrix}, \quad (\text{C10})$$

where $\varphi_n(y)$ is the n th harmonic oscillator ground state.

APPENDIX D: TUNNELING DENSITY OF STATES (TDOS)

1. Uniform case

The superconducting local TDOS for spin α for a uniform d -wave SC is given by

$$N_\alpha(\mathbf{r}, \Omega) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} [u_{\mathbf{k}}^2 \delta(\Omega - E_{\mathbf{k}}) + v_{\mathbf{k}}^2 \delta(\Omega + E_{\mathbf{k}})], \quad (\text{D1})$$

where $u_{\mathbf{k}}^2 = \frac{1}{2}(1 + \xi_{\mathbf{k}}/E_{\mathbf{k}})$, $v_{\mathbf{k}}^2 = \frac{1}{2}(1 - \xi_{\mathbf{k}}/E_{\mathbf{k}})$, and $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$. We can linearize the dispersion around the four nodes (labeled $\ell = \pm 1, \pm 2$), which leads to

$$\begin{aligned} N_\alpha(\mathbf{r}, \Omega) &= \sum_{\ell} \int^{\Lambda} \frac{d^2q}{(2\pi)^2} \frac{1}{2} \left[\left(1 + \frac{\vec{v}_f^{(\ell)} \cdot \vec{q}}{\mathcal{E}_q}\right) \delta(\Omega - \mathcal{E}_q) \right. \\ &\quad \left. + \left(1 - \frac{\vec{v}_f^{(\ell)} \cdot \vec{q}}{\mathcal{E}_q}\right) \delta(\Omega + \mathcal{E}_q) \right], \end{aligned} \quad (\text{D2})$$

where $\mathcal{E}_q = \sqrt{v_f^2 q_{\perp}^2 + v_{\Delta}^2 q_{\parallel}^2}$ and the momentum cutoff Λ ensures the same total number of momentum states. Doing the integral, we find

$$\begin{aligned} N_\alpha(\mathbf{r}, \Omega) &= 2 \int^{\Lambda} \frac{dq_{\parallel} dq_{\perp}}{(2\pi)^2} [\delta(\Omega - \sqrt{v_f^2 q_{\perp}^2 + v_{\Delta}^2 q_{\parallel}^2}) \\ &\quad + \delta(\Omega + \sqrt{v_f^2 q_{\perp}^2 + v_{\Delta}^2 q_{\parallel}^2})]. \end{aligned} \quad (\text{D3})$$

Rescaling $v_f q_{\parallel} = Q_1$ and $v_{\Delta} q_{\perp} = Q_2$, with $Q = \sqrt{Q_1^2 + Q_2^2}$, we find

$$N_\alpha(\mathbf{r}, \Omega) = 2 \int^{\Lambda} \frac{dQ}{2\pi v_f v_{\Delta}} Q [\delta(\Omega - Q) + \delta(\Omega + Q)] \quad (\text{D4})$$

with an appropriate choice $\Lambda = \sqrt{\pi v_f v_{\Delta}}$. Of course, this linearized description will break down at a lower energy

scale $\sim v_\Delta/a_0$, where a_0 is the lattice spacing. This yields, for $|\Omega| \lesssim v_\Delta/a_0 \ll \Lambda$,

$$N(\mathbf{r}, \Omega) = \sum_{\alpha} N_{\alpha}(\mathbf{r}, \Omega) = \frac{2|\Omega|}{\pi v_f v_\Delta}. \quad (\text{D5})$$

2. Pseudo-Landau level case: Gap variations

Consider the gap variation example discussed in the main text. Then, fermions at two of the Dirac points only see a phase change from the vector potential, which does not change the density of states, leading to a contribution from $\ell = \pm 2$ given by

$$N_2(\mathbf{r}, \Omega) = \frac{|\Omega|}{\pi v_f v_\Delta}. \quad (\text{D6})$$

This is half the total density of states in the uniform case. The contribution from the other node pair $N_1(\mathbf{r}, \Omega)$ is expected to reflect the formation of pseudo-LLs. The Green function for node pair $\ell = \pm 1$ reduces to

$$\begin{aligned} \mathcal{G}_{\alpha}^{(\ell=1)}(\mathbf{r}, i\Omega_m) &= \frac{1}{2L} \sum_k \left\{ \frac{\varphi_0^2(x - \frac{k}{\beta})}{i\Omega_m - E_0} + \frac{1}{2} \sum_{n>0} \left[\varphi_n^2 \left(x - \frac{k}{\beta} \right) \right. \right. \\ &\quad \left. \left. + \varphi_{n-1}^2 \left(x - \frac{k}{\beta} \right) \right] \left(\frac{1}{i\Omega_m - E_n} + \frac{1}{i\Omega_m + E_n} \right) \right\}, \end{aligned} \quad (\text{D7})$$

where $E_0 = 0$. Summing over spins and $\ell = \pm 1$, this leads to

$$\begin{aligned} N_1(\mathbf{r}, \Omega) &= \frac{2}{L} \sum_k \left\{ \varphi_0^2 \left(x - \frac{k}{\beta} \right) \delta(\Omega) + \frac{1}{2} \sum_{n>0} \left[\varphi_n^2 \left(x - \frac{k}{\beta} \right) \right. \right. \\ &\quad \left. \left. + \varphi_{n-1}^2 \left(x - \frac{k}{\beta} \right) \right] [\delta(\Omega - E_n) + \delta(\Omega + E_n)] \right\}. \end{aligned} \quad (\text{D8})$$

Deep in the bulk, $N_1(\mathbf{r}, \Omega)$ will be independent of \mathbf{r} , and we can approximate it as

$$N_1(\mathbf{r}, \Omega) \approx \frac{|\beta|}{\pi} \left[\delta(\Omega) + \sum_{n>0} [\delta(\Omega - E_n) + \delta(\Omega + E_n)] \right] \quad (\text{D9})$$

which can be recast in the more compact form

$$N_1(\mathbf{r}, \Omega) \approx \frac{|\beta|}{\pi} \sum_n \delta(\Omega - \lambda_n), \quad (\text{D10})$$

where $n = 0, \pm 1, \pm 2, \dots$, with $\lambda_n = \sqrt{2\beta v_f v_\Delta |n| \text{sgn}(n)}$. Thus, the total density of states $N_1(\mathbf{r}, \Omega) + N_2(\mathbf{r}, \Omega)$ will reflect a combination of the pseudo-LL spectrum as well as the Dirac density of states of the uniform d -wave SC.

APPENDIX E: d -WAVE SC IN A NARROW STRIP

In this appendix we study singular contributions to the TDOS which come from quantization of the quasiparticle momentum transverse to the strip. Just in this appendix, we find it convenient to retain the full BdG equation, and linearize around the Dirac nodes only at the end. We begin with the

BdG Hamiltonian

$$\hat{H}(k_L) = \begin{pmatrix} \xi(k_L, -i\partial_w) & \Delta(k_L, -i\partial_w) \\ \Delta(k_L, -i\partial_w) & -\xi(k_L, -i\partial_w) \end{pmatrix}, \quad (\text{E1})$$

where $0 < w < W$ is the transverse coordinate, and k_L, k_W will denote momenta along the strip length and strip width (L, W directions), respectively. For a (110) edge, we have $\xi(k_L, -k_W) = \xi(k_L, k_W)$ and $\Delta(k_L, -k_W) = -\Delta(k_L, k_W)$. We are looking for states which obey the strip boundary conditions, i.e., eigenfunctions $\psi(w)$ of \hat{H} which have a vanishing charge density at the strip edges $\psi^\dagger(0)\tau^z\psi(0) = \psi^\dagger(W)\tau^z\psi(W) = 0$. A plane-wave eigenfunction with positive eigenvalue $\varepsilon(k_L, k_W) = \sqrt{\xi^2(k_L, k_W) + \Delta^2(k_L, k_W)}$ is given by

$$\phi^+(k_L, k_W; w) = \begin{pmatrix} u(k_L, k_W) \\ v(k_L, k_W) \end{pmatrix} e^{ik_W w}, \quad (\text{E2})$$

where

$$|u(k_L, k_W)|^2 = \frac{1}{2} \left(1 + \frac{\xi(k_L, k_W)}{\varepsilon(k_L, k_W)} \right) \quad (\text{E3})$$

and

$$|v(k_L, k_W)|^2 = \frac{1}{2} \left(1 - \frac{\xi(k_L, k_W)}{\varepsilon(k_L, k_W)} \right). \quad (\text{E4})$$

Since $\Delta(k_L, k_W)$ is a real function for the d -wave SC we are considering, it is sufficient to take $u(k_L, k_W) > 0$ and $(\text{sign}v(k_L, k_W)) = (\text{sign}\Delta(k_L, k_W))$, thus, $u(k_L, -k_W) = u(k_L, k_W)$ and $v(k_L, -k_W) = -v(k_L, k_W)$. A plane-wave eigenfunction with negative energy $-\varepsilon(k_L, k_W)$ is given by

$$\phi^-(k_L, k_W; w) = \begin{pmatrix} v(k_L, k_W) \\ -u(k_L, k_W) \end{pmatrix} e^{ik_W w}. \quad (\text{E5})$$

To construct a state which obeys the boundary conditions, we consider a superposition of states with opposite k_W :

$$\begin{aligned} \psi^+(k_L, k_W > 0, w) &= \phi^+(k_L, k_W, w) + r(k_L, k_W) \phi^+(k_L, -k_W) \\ &= \begin{pmatrix} u(k_L, k_W) \\ v(k_L, k_W) \end{pmatrix} e^{ik_W w} + r(k_L, k_W) \begin{pmatrix} u(k_L, k_W) \\ -v(k_L, k_W) \end{pmatrix} e^{-ik_W w}. \end{aligned} \quad (\text{E6})$$

The charge density for this state is given by (dependence on k_L and k_W implicit)

$$\begin{aligned} \rho^+(w) &= \psi^{+\dagger}(w) \tau^z \psi^+(w) \\ &= u^2 [1 + |r|^2 + 2 \text{Re}(r e^{-i2k_W w})] \\ &\quad - v^2 [1 + |r|^2 - 2 \text{Re}(r e^{-i2k_W w})] \\ &= (u^2 - v^2)(1 + |r|^2) + 2 \text{Re}(r e^{-i2k_W w}), \end{aligned} \quad (\text{E7})$$

where $\text{Re}(z)$ denotes the real part of z . Finite-size quantization sets as usual $k_W = \pi n/W$, where $n = 0, 1, 2, \dots$, while demanding that ρ^+ vanish at the strip edges results in

$$(u^2 - v^2)(1 + |r|^2) + 2 \text{Re}(r) = 0. \quad (\text{E8})$$

Since $-1 \leq u^2 - v^2 \leq 1$, r is always real. Thus, the eigenstates are given by

$$\begin{aligned} \psi_n^+(k_L; w) &= \begin{pmatrix} u_n(k_L) \\ v_n(k_L) \end{pmatrix} e^{i\pi n w / W} \\ &+ r_n(k_L) \begin{pmatrix} u_n(k_L) \\ -v_n(k_L) \end{pmatrix} e^{-i\pi n w / W}. \end{aligned} \quad (\text{E9})$$

Similar states with negative energy are given by

$$\begin{aligned} \psi_n^-(k_L; w) &= \begin{pmatrix} v_n(k_L) \\ -u_n(k_L) \end{pmatrix} e^{i\pi n w / W} \\ &+ r_n(k_L) \begin{pmatrix} -v_n(k_L) \\ -u_n(k_L) \end{pmatrix} e^{-i\pi n w / W}. \end{aligned} \quad (\text{E10})$$

The TDOS is given by

$$\begin{aligned} N(\Omega, w) &= \frac{1}{2} \int \frac{dk_L}{2\pi} \sum_{n=0}^{\infty} \sum_{s=\pm} \psi_n^{s\dagger}(k_L, w) (\tau^0 + s\tau^z) \psi_n^s(k_L, w) \\ &\times [\Omega - s\varepsilon_n(k_L)]. \end{aligned} \quad (\text{E11})$$

Focusing on positive energies,

$$\begin{aligned} N(\Omega > 0, w) &= 2 \int \frac{dk_L}{2\pi} \sum_n u_n^2(k_L) \left(1 + r_n^2(k_L) \right. \\ &\left. + 2r_n(k_L) \cos \frac{\pi n w}{W} \right) \delta[\Omega - \varepsilon_n(k_L)]. \end{aligned} \quad (\text{E12})$$

The main low-energy contributions to the TDOS in a d -wave SC come from the vicinity of the nodes. We are further focusing on the nodes at $k_L = 0$ and $k_W = \pm K_F$, thus, for $k_W > 0$, $\xi \simeq v_f(n\pi/W - K_F)$, $\Delta \simeq v_\Delta k_L$, and $\varepsilon \simeq \sqrt{v_f^2(n\pi/W - K_F)^2 + v_\Delta^2 k_L^2}$. Changing integration variables we have

$$\begin{aligned} N(\Omega > 0, w) &= 2 \int_{v_f|n\pi/W - K_F|}^{\infty} \frac{\varepsilon d\varepsilon}{2\pi} \sum_n \frac{1}{v_\Delta k_n(\varepsilon)} u_n^2(\varepsilon) \\ &\times \left(1 + r_n^2 + 2r_n \cos \frac{\pi n w}{W} \right) \delta(\Omega - \varepsilon) \\ &= \sum_n \Theta(\Omega - v_f|n\pi/W - K_F|) \frac{\Omega}{\pi} \frac{1}{v_\Delta k_n(\Omega)} u_n^2(\Omega) \\ &\times \left(1 + r_n^2 + 2r_n \cos \frac{\pi n w}{W} \right), \end{aligned} \quad (\text{E13})$$

where $k_n(\Omega) = \sqrt{\Omega^2 - (v_f n\pi/w - K)^2}/v_\Delta$ and $u_n^2(\Omega) = [1 + v_f(n\pi/w - K_F)/\Omega]/2$. Since there are always values of w for which the term in the above parentheses is finite, we find that there are contributions at $\Omega = v_f|n\pi/W - K_F|$ which diverge as $1/\sqrt{\Omega^2 - (v_f n\pi/W - K_F)^2}$.

APPENDIX F: PSEUDO-LANDAUE LEVELS OF STRAINED d -WAVE SC IN THE (1,0)-EDGED STRIP GEOMETRY

Numerical diagonalization of the lattice BdG Hamiltonian was also performed for a (1,0)-edged strip. Again, the strip's width is W , and the transverse direction, along which periodic boundary conditions were used, has length $L \gg W$.

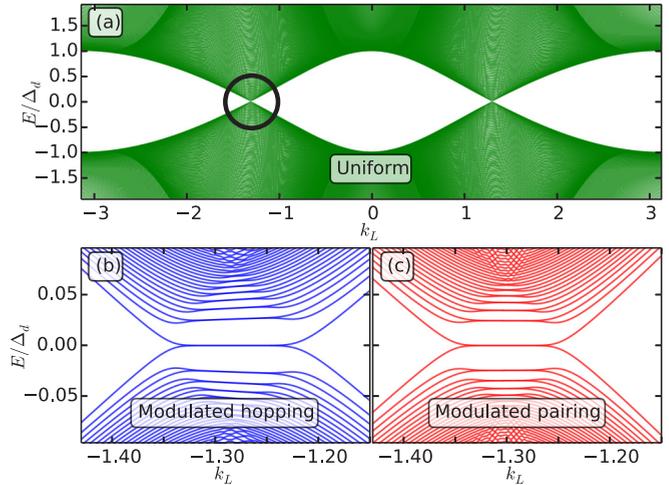


FIG. 5. (a) Spectrum of uniform d -wave SC on a (1,0)-edged strip versus momentum k_L along the L direction showing Dirac nodes. Note that there are no zero-energy ABSs in this geometry. Circle indicates region shown in the next two panels. (b) Formation of flat pseudo-Landau levels in the low-energy regime due to uniform hopping-amplitude gradient in the [1,0] direction; shown here is the near-node region indicated in (a). (c) Similar to (b) but with extended s -wave pairing gradient.

Parameters t , t' , \bar{n} , and Δ_d are taken to be the same as in the (1,1)-edged case considered in the main text.

Figure 5(a) shows the spectrum of the strip as a function of the momentum k_L along the long direction L in the absence of any imposed spatial variation. The spectrum exhibits the d -wave Dirac nodes projected onto the Brillouin zone of the strip. As expected with (1,0) edges, zero-energy ABSs are absent from the spectrum. A circle indicates the near-node region in which we have chosen to plot the spectra of Figs. 5(b) and 5(c).

Figure 5(b) shows the spectrum in the presence of a nonzero gradient in the hopping amplitude across the strip width (in the [1,0] direction), which leads to a pseudo-LL spectrum at both Dirac nodes; we have chosen $W = 3000a_0$ and a maximum change $\delta t \sim 0.25t$ at the edge. Figure 5(c) shows the effect of an extended s -wave pairing gradient across the strip width, also leading to pseudo-LL formation at both Dirac nodes. Here, we have chosen $W = 3000a_0$ and a maximum s -wave gap $\Delta_s \sim 0.25\Delta_d$ at the edge. The low-energy spectra in Figs. 5(b) and 5(c) are in quantitative agreement with our analytical results.

APPENDIX G: MEAN FIELD EQUATIONS FOR CORRELATED d -WAVE SC WITH STRAIN

We start from the usual tJ model in the main text

$$H_{tJ} = -g_t \sum_{i,j,\alpha} t_{0,ij} c_{i\alpha}^\dagger c_{j\alpha} + g_J J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j, \quad (\text{G1})$$

where the *bare* nearest-neighbor and next-neighbor hoppings are $t_0 = 1$ and $t'_0 = -0.3t_0$, respectively, the antiferromagnetic exchange coupling $J = 4t_0^2/U = 0.3t_0$, and the renormalization factors $g_t = 2p/(1+p)$, $g_J = 1$ account for strong correlation effects in a mean field manner. Note that g_t is chosen in line with renormalized mean field theory, while we have set $g_J = 1$ similar to what one expects from slave-boson

mean field theory. At any rate, we should only view this as an effective model to obtain a variational d -wave superconducting ground state, with results which approximately reproduce experimental data. Doing a full Hartree-Fock-Bogoliubov mean field theory of the superexchange term, we arrive at the mean field Hamiltonian

$$H_{\text{MFT}} = \sum_{\mathbf{k}\alpha} \xi_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha} - \sum_{\mathbf{k}} \Delta_{\mathbf{k}} (c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger + c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}), \quad (\text{G2})$$

where $\xi_{\mathbf{k}} = -2(g_t t_0 + \frac{3}{4} g_J J \chi)(\cos k_x + \cos k_y) - 4g_t t_0' \cos k_x \cos k_y$ is set by the effectively renormalized hoppings (which appear in our BdG calculations in the paper) $t = (g_t t_0 + \frac{3}{4} g_J J \chi)$ and $t' = g_t t_0'$, while the pairing gap $\Delta_{\mathbf{k}} = \frac{3}{2} g_J J \Delta_0 (\cos k_x - \cos k_y)$. The mean field equations determining χ, Δ_0 and the mean electron density $\bar{n} \equiv 1 - p$ are given by

$$\Delta_0 = \frac{1}{2N} \sum_{\mathbf{k}} \frac{\Delta_{\mathbf{k}}}{2E_{\mathbf{k}}} (\cos k_x - \cos k_y), \quad (\text{G3})$$

$$\chi = \frac{1}{4N} \sum_{\mathbf{k}} \left(1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right) (\cos k_x + \cos k_y), \quad (\text{G4})$$

$$\bar{n} = \frac{1}{N} \sum_{\mathbf{k}} \left(1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right) \equiv 1 - p, \quad (\text{G5})$$

where $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$. We solve these equations self-consistently assuming $t_0 \rightarrow t_0(1 + \varepsilon)$ and $J \rightarrow J(1 + 2\varepsilon)$, where the (small) fractional change ε in the hopping and exchange interaction is determined by the strain which affects the lattice constant; see main text. (The factor of 2ε in J reflects its dependence on hopping as $\sim t_0^2$.)

We pick the bare hopping $t_0 = 450$ meV, which leads to $J = 135$ meV (corresponding to $U/t_0 \approx 13$). For hole doping $p = 0.15$, and for the unstrained case $\varepsilon = 0$, we find that the renormalized hoppings satisfy $t' = -0.25t$, and an antinodal gap $3g_J J \Delta_0 \approx 24$ meV at $(\pi, 0)$. In addition, with the lattice constant $a_0 = 3.85$ Å, we find a nodal Fermi velocity $v_f \approx 1.3$ eV Å, and a ratio of Fermi velocity to gap velocity $v_f/v_\Delta \approx 20$. These are in reasonable agreement with results for the optimally doped cuprates. Incorporating ε , and solving the mean field equations, we find the results for the strain dependence of the hopping and pairing quoted in the main text.

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