UNIVERSITY OF TORONTO
Faculty of Applied Science and Engineering

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PHY293F (Waves and Modern Physics)
Instructor: Professors N. Grisouard and W. Trischuk

Duration: 2.5 hours

Exam Type A: Closed Book. Only non-programmable calculators allowed.

This examination paper consists of **6** pages and 6 questions. Please bring any discrepancy to the attention of an invigilator. Answer all 6 questions.

- Write your name, student number and tutorial group on top of **all** examination booklets and test pages used.
- Aids allowed: only calculators, from a list of approved calculators as issued by the Faculty Registrar are allowed. Not other aid (notes, textbook, dictionary) is allowed. **Communication devices are strictly forbidden. Turn them off and make sure they are in plain sight to the invigilators.**
- Each question is worth 1/6 of your overall grade for this exam. Within each question, a mark breakdown is indicated in square brackets at the end of each sub-part. Part marks will be given for partially correct answers, so show any intermediate calculations that you do and write down, **in a clear fashion** any relevant assumptions you are making along the way.
- Do not separate the stapled sheets of the question paper. Hand in the questions with your exam booklet at the end of the test.
- The next two pages include some formulae and constants you may find useful.
- The questions begin on **page 4**. The total number of marks is 60.

Oscillations

	Amplitude	Velocity	Dissipated Power
Peak freq.	$\omega_{max} = \omega_0\sqrt{1 - 1/(2Q^2)}$	$\omega_{max} = \omega_0$	$\omega_{max} = \omega_0$
Peak value	$A_{max} = \dfrac{QA_f}{\sqrt{1 - 1/(4Q^2)}}$	$V_{max} = \omega_0 Q A_f$	$P_{max} = \dfrac{mA_f^2\omega_0^3 Q}{2}$
Misc.	$A(\omega) = \dfrac{\omega_0^2 A_f}{\sqrt{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}}$	$V(\omega) = \omega A(\omega)$	$\overline{P}(\omega) = \dfrac{m\gamma V^2(\omega)}{2}$
	$\tan\delta = \dfrac{\omega\gamma}{\omega_0^2 - \omega^2}$		$\approx \dfrac{P_{max}}{1 + \frac{4(\omega_0-\omega)^2}{\gamma^2}}\ (Q \gg 1)$

$$\mathsf{M}\ddot{\vec{X}} + \mathsf{K}\vec{X} = 0; \qquad \det\left(\mathsf{K} - \omega^2\mathsf{M}\right) = 0.$$

$$\mathsf{M}^{-1}\mathsf{K} \quad \text{symmetric and} \quad |\vec{Y}_i| = 1 \quad \Rightarrow \quad \vec{Y}_i \cdot \vec{Y}_j = \delta_{ij}$$

$$\vec{X}(t) = \sum_{n=1}^{N} C_n \vec{Y}_n \cos(\omega_n t + \phi_n), \quad \text{with} \quad C_n\cos\phi_n = \vec{X}_0 \cdot \vec{Y}_n \quad \text{and} \quad C_n \sin\phi_n = -\frac{\vec{V}_0 \cdot \vec{Y}_n}{\omega_n}.$$

$$\det\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

$$ax^2 + bx + c = 0 \qquad \Rightarrow \qquad x = \frac{1}{2a}\left(-b \pm \sqrt{b^2 - 4ac}\right)$$

$$\frac{\partial^2}{\partial t^2}y(x,t) - v^2\frac{\partial^2}{\partial x^2}y(x,t) = 0 \quad \text{with} \quad v = \sqrt{\frac{T}{\mu}}$$

$$y(x,t) = \sum_{n=1}^{\infty} C_n \cos(\omega_n t + \phi_n)\sin(k_n x) = \sum_{n=1}^{\infty}[\alpha_n\cos(\omega_n t) + \beta_n \sin(\omega_n t)]\sin(k_n x),$$

$$\text{with} \quad \alpha_n = \frac{2}{L}\int_0^L y(0,x)\sin(k_n x)\mathrm{d}x \quad \text{and} \quad \beta_n = \frac{2}{L\omega_n}\int_0^L \dot{y}(0,x)\sin(k_n x)\mathrm{d}x.$$

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x - vt)\right) = A\sin\left(k(x - vt)\right) = A\sin\left(kx - \omega t\right) = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right].$$

$$\omega = 2\pi\nu, \quad \nu = 1/T, \quad k = 2\pi/\lambda, \quad v = \omega/k = \lambda/T = \lambda\nu.$$

$$\text{Energy Flux} = \frac{1}{2}\mu_i v\omega^2 A^2 = \frac{1}{2}\sqrt{T\mu_i}\omega^2 A^2.$$

$$\rho = \frac{A_R}{A_I} = \frac{\sqrt{\mu_1} - \sqrt{\mu_2}}{\sqrt{\mu_1} + \sqrt{\mu_2}}; \quad \tau = \frac{A_T}{A_I} = \frac{2\sqrt{\mu_1}}{\sqrt{\mu_1} + \sqrt{\mu_2}}$$

Modern Physics

Speed of light $c = 3.00 \times 10^8$ m/s

Elementary charge $e = 1.602 \times 10^{-19}$ C

Coulomb constant $\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C}^2/(\mathrm{J\,m})$

$hc = 1.240$ keV nm

Mass of electron $m_\mathrm{e} = 9.11 \times 10^{-31}$ kg $= 511$ keV/c^2

Mass of proton $m_\mathrm{p} = 1.67 \times 10^{-27}$ kg $= 939$ MeV/c^2

Planck's constant $h = 6.626 \times 10^{-34}$ J s $= 4.14 \times 10^{-15}$ eV s

$1\,\mathrm{eV} = 1.602 \times 10^{-19}$ J

$$L^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 \qquad \gamma = (1-\beta^2)^{-1/2} \qquad \beta = v/c$$

$$x' = \gamma\,(x - \beta\,ct) \qquad ct' = \gamma\,(ct - \beta\,x) \qquad y' = y \qquad z' = z$$

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2}$$

$$\vec{p} = m\,\vec{u} \qquad \vec{p} = \gamma\,m\,\vec{u}$$

$$E = \gamma\,mc^2 = \sqrt{(pc)^2 + (mc^2)^2} \qquad \left(\frac{p}{mc}\right)^2 = \gamma^2\beta^2$$

$$E = h\nu = \hbar\omega \qquad \lambda = \frac{h}{p} \qquad \vec{p} = \frac{h}{\lambda} = \hbar\vec{k} \qquad \hbar = \frac{h}{2\pi}$$

$$T_\mathrm{max} = eV_0 = h\nu - \Phi \qquad \Delta\lambda = \lambda_\mathrm{f} - \lambda_\mathrm{i} = \frac{h}{mc}\,(1 - \cos\theta)$$

$$E_n = -\frac{1}{2n^2}\,m_\mathrm{e}\Big(\frac{e^2}{4\pi\epsilon_0\hbar}\Big)^2 = \frac{E_1}{n^2} = -\frac{13.56\,\mathrm{eV}}{n^2}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x,t)\,\Psi(x,t) = E\,\Psi(x,t) = \mathrm{i}\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

$$\int_\text{all space} \big|\Psi(x,t)\big|^2\,dx = 1 \qquad \Delta p\,\Delta x \gtrsim \hbar/2 \qquad \Delta E\,\Delta t \gtrsim \hbar/2$$

$$\cos\theta = \frac{\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta}}{2} \qquad \sin\theta = \frac{\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}}{2\mathrm{i}} \qquad \begin{Bmatrix}\cos^2\theta \\ \sin^2\theta\end{Bmatrix} = \frac{1}{2}\,(1 \pm \cos\,2\theta)$$