UNIVERSITY OF TORONTO
Faculty of Applied Science and Engineering

Second Term Test: 2017 Solutions [grades]

PHY293F (Modern Physics)
Instructor: Professor W. Trischuk

Duration: 1 hour

Exam Type C: Non-programmable calculators.

This examination paper consists of **3** pages and 3 questions. Please bring any discrepancy to the attention of an invigilator. Answer all 3 questions.

- This page includes some formulae and constants you may find useful.
- The questions are on **page 2**.
- Each question is worth 1/3 of your overall grade for this term test. Within each question, sub-parts of the question will be weighted equally. Part marks will be given for partially correct answers, so show any intermediate calculations that you do and write down, **in a clear fashion** any relevant assumptions you are making along the way.

Possibly Useful Formulae and Constants

$c = 3.00 \times 10^8$ m/s	$\gamma = 1/\sqrt{1-\frac{v^2}{c^2}}$	$x' = \gamma(x - vt)$
$x = \gamma(x' + vt')$	$t = \gamma(t' + vx'/c^2)$	$t' = \gamma(t - vx/c^2)$
$\lambda_{source} = \lambda_{observer}\frac{\sqrt{1-v^2/c^2}}{1+(v/c)\cos\theta}$	$u'_x = \frac{u_x - v}{1-u_x v/c^2}$	$u'_y = \frac{u_y}{\gamma(1-u_x v/c^2)}$
$E^2/c^2 - \vec{p}\cdot\vec{p} = m^2c^2$	$E = \gamma mc^2$	$p_x = \gamma m u_x$

1. Anna is at the origin of a stationary inertial reference frame and observes a firecracker explode 100 m away from her along the x-axis, at a time $t = 1.0$ μs. Bob is at the origin of another inertial reference frame that is moving at $v = \frac{5}{13}c$ along the x-axis of Anna's frame. The origin of the two coordinate systems coincide (ie. $x = x' = 0$) at $t = t' = 0$.

   (a) At what position (x') and at what time (t') does Bob observe the firecracker explosion? $v = 5/13c \rightarrow \gamma = 13/12$. Use the Lorentz transformations to find the coordinates in Bob's frame: $x' = \gamma(x - vt) = -16.6$m and $t' = \gamma(t - vx/c^2) = 0.94$ μs. [2.5]. Bob will actually be 16.6 m **past** the point where he sees the firecracker explode, and it will happen 0.94 μs after he passed Anna, according to clocks in his reference frame.

   (b) Compute the spacetime interval, in Anna's frame, between the firecracker explosion and a second 'event' which occurs when the two coordinate systems coincide. The spacetime interval is $I = (ct)^2 - x^2 = (3 \times 10^8 \cdot 1 \times 10^{-6})^2 - 100^2 = 80000$ m^2 in Anna's frame [2.5]

   (c) What should Bob measure as the spacetime interval between these two events in his frame. It is not necessary to provide a calculation to answer this part of the question. Bob will measure the same spacetime interval between the two events, as it is frame independent. So no calculation is necessary. Bob will also see $I' = 80000$ m^2. [2.5] You could calculate this from the answers in part b), but you'd have to keep more significant figures than I did.

   (d) Given the sign of the spacetime interval what can you say about any causal connection between Bob and Anna's meeting at the origin (the second event in part b)) and the explosion of the firecracker? This spacetime interval is positive, so it is time-like. That means there can be a causal connection between Anna and Bob's meeting when their respective origins coincide and the explosion of the firecracker [2.5]. Indeed Anna sees the firecracker explode 1 μs after her meeting with Bob, somewhere along the positive x direction, while Bob sees it explode somewhat less time after their meeting, but in the negative x direction. The interval is time-like, so Bob and Anna (and all other inertial observers) will agree that the meeting takes place before the firecracker explodes.

2. A light source, at rest, emits light at a wavelength of 532 nm.

   (a) If this source moves along a line that connects it with the Earth, and observers on Earth see light at 670 nm what is the source's velocity? You should clearly indicate both the speed, and direction of travel along this line that connects the source to the Earth. You are given the Doppler shift formula on the front of the exam. If the source is moving 'away' then the light will be red-shifted ... shifted to longer wavelengths. This is what is actually seen here, so we can put $\theta = 0$ in to the formula given and simplify to get $\frac{\lambda_s}{\lambda_o} = \sqrt{\frac{1-\beta}{1+\beta}}$ Note that the formula given is for the wavelengths (not the frequencies) but that **is** the one you need for this problem, I was trying to be nice. You can re-arrange this, if x is the ratio of the wavelengths (source/observed) then $\beta = \frac{x^2-1}{x^2+1} = 0.227$ for the two wavelengths given in this problem. So the source is receding from the earth and $v = 0.227c$. [5]. Note that you don't need to start with the understanding that the source is receding. If you tried to assume the source was approaching ($\theta = 180^o$)

you would find that the ratio (source/observed) would have to be larger than 1 and then you'd have to try it 'this way'. I think you had time to try both in answering this question ... if you needed it.

(b) If it were to move in the other direction but at the same speed, what wavelength would be seen on Earth? If the source were approaching Earth then we'd have $theta = 180^o$ and $\lambda_s/\lambda_o = \sqrt{\frac{1+\beta}{1-\beta}}$ or $\lambda_o = \lambda_s\sqrt{\frac{1-\beta}{1+\beta}}$. Given the 532 nm source wavelength and $\beta = 0.227$ this gives 422 nm observed. [5]

3. A charged particle is accelerated, from rest, through an electric potential difference of V volts. After it has passed through the electric potential difference it has a speed of $v = 0.998c$ Accelerating a charged particle through a potential difference adds kinetic energy. But when it is up to speed the particle's total energy is given by $E = \gamma mc^2$. So the kinetic energy change is $\Delta E_{kin} = (\gamma - 1)mc^2$. $\gamma = 1/\sqrt{1 - v^2/c^2}$ from the formula sheet and is $\gamma = 15.8$ for both particles in this problem. [2+2].

(a) What potential difference is necessary, if the particle is an electron with mass $511\ \mathrm{keV}/c^2$? For the electron the kinetic energy change is 14.8×511 keV or 7.6 MeV. This means the electron must be accelerated through a 7.6 MV (million volt) potential difference [2].

(b) What potential difference is necessary if the particle is a proton with mass $938\ \mathrm{MeV}/c^2$? For the proton the kinetic energy change is 14.8×938 MeV or 13.9 GeV. This means the electron must be accelerated through a 13.9 GV (billion volt) potential difference [2]. In practice this is usually done by accelerating a proton in a circular accelerator and passing it through a single few MV potential difference 1000 to 10000 times.

(c) What is the total momentum of such an electron and proton, respectively? Just use $p = \gamma mv$ from the formula given on the front of the test. For the electron this gives: $15.8 \times 0.511\mathrm{keV/c^2} \times 0.998c = 8.06\mathrm{MeV}/c$ [1]. And for the proton this gives $15.8 \times 0.938\mathrm{MeV/c^2} \times 0.998c = 14.8\mathrm{GeV}/c$. [1].

End of examination
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