Lecture notes from Prof. Peter Krieger

You are not responsible for this material on the final exam in PHY293 in 2017.

Prof. Trischuk

PHY293B (Modern Physics)

Lectures 18

Interpretations of Quantum Mechanics

Interpretations of Quantum Mechanics

[This discussion follows section 1.2 of D.J. Griffiths text on Quantum Mechanics along with his afterword in chapter 12 (for Bell's Theorem) See also Sec. 4.5 of your text where the Copenhagen interpretation is discussed briefly.]

If you measure the location of some subatomic particle (i.e. a particle described by a wavefunction Ψ) and find it at some point c, where was it before you made the measurement?

There are three plausible responses to this:

- It was at c
- It was nowhere in particular
- It doesn't matter – the question does not really make sense

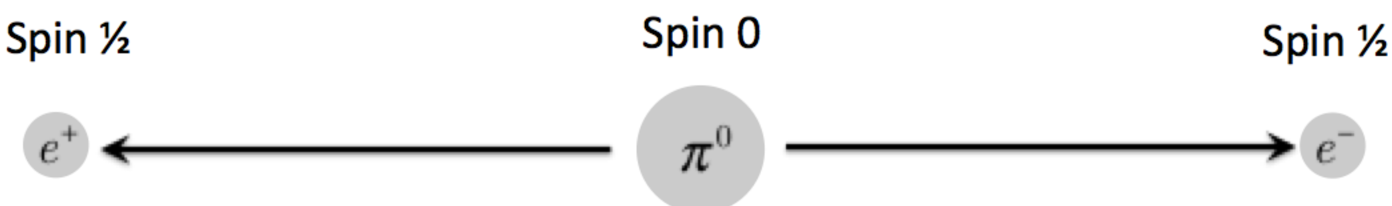
Let's take a closer look at the options:

Interpretations of Quantum Mechanics

- The **realist** position: the particle was at c
  - If this is the case than QM must be an incomplete theory, since the particle was at c but the theory is unable to predict this. This implies the existence of some variable that is "hidden" to the theory (i.e. to Quantum Mechanics).
- The **orthodox** position: it wasn't anywhere. It was everywhere where $\Psi(x,t) \neq 0$ with probability density $\Psi^{*}\Psi$ until a measurement forced it to be "somewhere" (i.e. collapsed the wavefunction):
  - This is the so-called Copenhagen or statistical interpretation of QM
- The **agnostic** position: refuse to answer on the grounds that it doesn't matter. What does it really mean to ask where a particle was before you measured it's position? You need to make a measurement to test your hypothesis, and that measurement will always change the state.
- For a long time this was a purely philosophical debate

John Bell

- In 1964, John Bell showed that it actually makes an observable difference whether or not a particle has a precise (but unknown) value for some observable, prior to a measurement.
- This makes the issue of whether answer the realist or orthodox position is correct an **experimental** question, rather than a purely theoretical one.
- The agnostic position then also becomes untenable.
- For this discussion, we first need to know about the EPR paradox.



Spin (here for electrons)

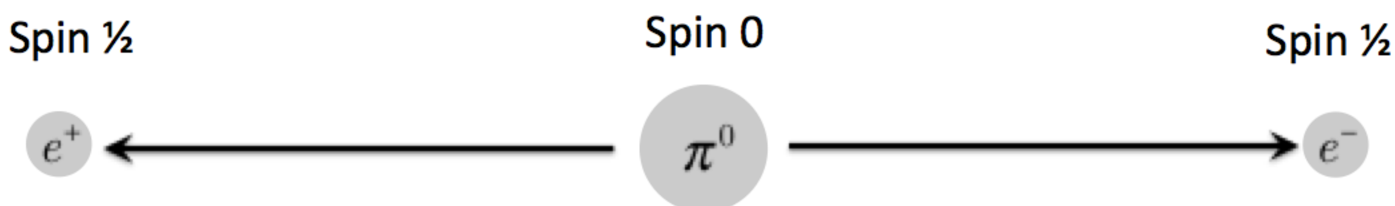
- Spin is a property of fundamental particles. It is a kind of intrinsic angular momentum.
- Electrons have a spin of ½ (in units of ħ)
- Spin makes a particle a little like a small bar magnet. The spin will tend to align with an applied magnetic field.
- When you measure the spin of an electron you measure it along some axis:
  - The only possible values for a measurement along any axis are ±½
  - These two outcomes are sometimes referred to as spin-up and spin-down

[Here I've told you only as much as we need to know for the coming discussion.]

The Einstein-Podolsky-Rosen (EPR) Paradox

[See Griffths QM Chapter 12]

- The EPR paradox was first published in 1935 and was intended to support the realist view of QM (which Einstein et al. all adhered to).
- Consider the decay of a neutral pion to an electron-positron pair:



For this you just need to know that since the pion has spin-0, the spins of the electron and positron must be opposite. So if one is "spin-up" the other must be "spin-down". (This is just conservation of angular momentum).

There are thus two possible spin configurations for the final-state electron-positron pair: $\uparrow_+ \downarrow_-$ or $\downarrow_+ \uparrow_-$

The Einstein-Podolsky-Rosen (EPR) Paradox

Quantum mechnically, the system exists in a linear superposition of the two possible states, prior to a measurement being made.

The wavefunction describing the spin-state is thus

$$\psi_{spin} = \frac{1}{\sqrt{2}}\left(\uparrow_{+}\downarrow_{-} - \uparrow_{-}\downarrow_{+}\right)$$

The details are not critical here: the important issue is that there are two possible states of the system and, before a measurement, the system has a wavefunction that is a linear superposition of the two.

Quantum mechanics cannot tell you e.g. what a measurement of the spin of the electron (e^{-}) will yield, only that it will be one of two possible values, with equal probability.

However, making the measurement collapses the wavefunction of the entire system. If we measure $\uparrow$ for the electron then a measurement of the positron spin **must** yield $\downarrow$.

The EPR Paradox

- Suppose we let the electron and positron fly very far apart, then we measure the electron spin. We immediately collapse the wavefunction and force the positron into a particular state, even though it may be many light-years away.
- The wavefunction collapse is instantaenous!
- In the realist view this is OK, because the electron and positron always had whatever spins they had. We just didn't know what they were until we made a measurement.
- In the orthodox (Copenhagen) interpretation, the measurement had an effect on particle that was light-years away. Einstein famously referred to this as "*spooky action at a distance*" and found the idea preposterous.
- This violates the principle of "locality".

The EPR Paradox

- The basis of the paradox, of course, is that nothing travels faster than the speed of light.
- But if the wavefunction collapse is not instantaneous (e.g. if it somehow propagates with a finite speed) we could violate angular momentum conservation by measuring the the positron spin after we measured the electron spin, but before the wavefunction collapse "caught up with" the positron. This does not happen.
- This pair of particle is an example of what is known as an **entangled state**. Such states are still the topic of intensive research in QM.
  - They are very important e.g. for quantum cryptography, quantum information, quantum computing, quantum teleportation etc.
- Einstein, Podolsky and Rosen did not believe QM was wrong, only that is was incomplete, i.e. that there is some variable that is "hidden" from the theory (such theories are known as "hidden variable theories").

John Bell (1964)

- John Bell devised a measurement for which the predictions of QM alone and QM + hidden-variable theory are very different.
- Imagine we let the electrons in the EPR setup fly far apart, then measure the spin of each, along the same axis. Let's just use ±1 for the possible results rather than ±½ (this won't matter).
  - The results will ALWAYS be of opposite sign. So the product is always −1.
- Bell's suggestion was the following: instead of having the two detectors measure along the same axis, have the two axes be randomly oriented. So one detector (A) measures the component of the spin along some direction (unit vector **a**) and the other (B) along some direction (unit vector **b**).
- For a large number of π^0 decays, one then measures the product of the two measurements (which can now be either +1 or −1).

Bell proposed to measure the average value of this product over a large number of decays: let's call this P(**a**,**b**). For **a**=**b** we recover the original case. That is P(**a**,**a**)= -1 (this is of course also true of P(**b**,**b**)).

If the two axes are anti-parallel, then one always gets +1 (P(**a**,-**a**)= +1).

For arbitrary orientations, QM predicts P(**a**,**b**)= $-\mathbf{a}\cdot\mathbf{b}$

What Bell did was to show that this result is incompatible with any hidden-variable theory.

Assume that the complete state of the electron/positron system is characterized by the wavefunction PLUS, some hidden variable λ (in some way that we do not know or have any control over from one pion decay to the next). We will just write things in terms of λ below.

We will also assume that the two detectors are not affected by one another. Then there must be some function $A(\mathbf{a},\lambda)$ that dictates the outcome of the measurement of detector A and an equivalent function $B(\mathbf{b},\lambda)$. These functions can only take the values:

$$A(\mathbf{a},\lambda) = \pm 1,\ B(\mathbf{b},\lambda) = \pm 1$$

If the detectors are aligned ($\mathbf{a}=\mathbf{b}$) we have $A(\mathbf{a},\lambda) = -B(\mathbf{a},\lambda)$ for all λ.

The average of the product of the measurements (which is what we are interested in) is:

$$P(\mathbf{a},\mathbf{b}) = \int \rho(\lambda)\, A(\mathbf{a},\lambda) B(\mathbf{b},\lambda)\, d\lambda$$

Where $\rho(\lambda)$ is the the probability density for the hidden variable which must satisfy the usual requirements on a probability density, e.g. it is non-negative and satisfies

$$\int_{-\infty}^{\infty} \rho(\lambda)\, d\lambda = 1.$$

The expression at the bottom of slide 11 [$A(\mathbf{a},\lambda) = -B(\mathbf{a},\lambda)$] allows us to write

$$B(\mathbf{b},\lambda) = -A(\mathbf{b},\lambda)$$

Which leads to

$$P(\mathbf{a},\mathbf{b}) = -\int \rho(\lambda)\, A(\mathbf{a},\lambda) A(\mathbf{b},\lambda)\, d\lambda.$$

$$P(\mathbf{a},\mathbf{b}) = -\int \rho(\lambda)\, A(\mathbf{a},\lambda)A(\mathbf{b},\lambda)\; d\lambda$$

If **c** is any other unit vector we can write:

$$P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{c}) = -\int \rho(\lambda)[A(\mathbf{a},\lambda)A(\mathbf{b},\lambda) - A(\mathbf{a},\lambda)A(\mathbf{c},\lambda)]\, d\lambda$$

Since $[A(\mathbf{b},\lambda)]^2 = 1$, this can be rewritten as:

$$P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{c}) = -\int \rho(\lambda)[1 - A(\mathbf{b},\lambda)A(\mathbf{c},\lambda)]A(\mathbf{a},\lambda)A(\mathbf{b},\lambda)\, d\lambda$$

We had $A(\mathbf{a},\lambda) = \pm 1$, $B(\mathbf{b},\lambda) = \pm 1$ which implies:

$-1 \leq A(\mathbf{a},\lambda)A(\mathbf{b},\lambda) \leq +1$ and we also know that $\rho(\lambda)[1 - A(\mathbf{b},\lambda)A(\mathbf{c},\lambda)] \geq 0$

So we have: $|P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{c})| \leq \int \rho(\lambda)[1 - A(\mathbf{b},\lambda)A(\mathbf{c},\lambda)]\, d\lambda$

We just had: $|P(\mathbf{a},\mathbf{b})-P(\mathbf{a},\mathbf{c})| \leq \int \rho(\lambda)[1-A(\mathbf{b},\lambda)A(\mathbf{c},\lambda)]\,d\lambda$

$$|P(\mathbf{a},\mathbf{b})-P(\mathbf{a},\mathbf{c})| \leq \underbrace{\int \rho(\lambda)\,d\lambda}_{=1} \underbrace{- \int \rho(\lambda)[A(\mathbf{b},\lambda)A(\mathbf{c},\lambda)]\,d\lambda}_{P(\mathbf{b},\mathbf{c})}$$

$$|P(\mathbf{a},\mathbf{b})-P(\mathbf{a},\mathbf{c})| \leq 1+P(\mathbf{b},\mathbf{c})$$

Since I was asked about this after the lecture, let me point out that is 1+P(**b**,**c**) because the negative sign (above) is part of the definition of P(**b**,**c**). See the top of the previous slide.

This is know as **Bell's Inequality**. It must be satisfied by any hidden variable theory (that is, any theory that says the the spins of each of the two electrons are fixed when they are produced).

Note that we have made no assumptions about ρ(λ) except those that apply to any probability density.

It is not hard to show that the prediction of Quantum Mechanics do not obey this.

Suppose the three vectors **a**, **b**, **c** lie in a plane with **a** and **b** perpendicular to one another, and **c** at 45° to each of them.

For this case, QM says: $P(\mathbf{a},\mathbf{b}) = 0$,
$P(\mathbf{a},\mathbf{c}) = P(\mathbf{b},\mathbf{c}) = -0.707$

This is incompatible with Bell's Inequality, since:

$$|P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{c})| \leq 1 + P(\mathbf{b},\mathbf{c})$$

$$0 - (-0.707) \overset{?}{\leq} 1 - 0.707$$

$$0.707 \overset{?}{\leq} 0.293 \quad ✗$$

This means that hidden-variable theories and QM are INCOMPATIBLE. Which one is correct can be determined from experiment (by making the proposed measurements).

Such experiments have been done in many different ways since the 1960s and 1970s.

You can do one such experiment in the 3rd and 4th year Advance Undergraduate Physics Lab, using entangled photons.

The results of such experiments are unequivocal:

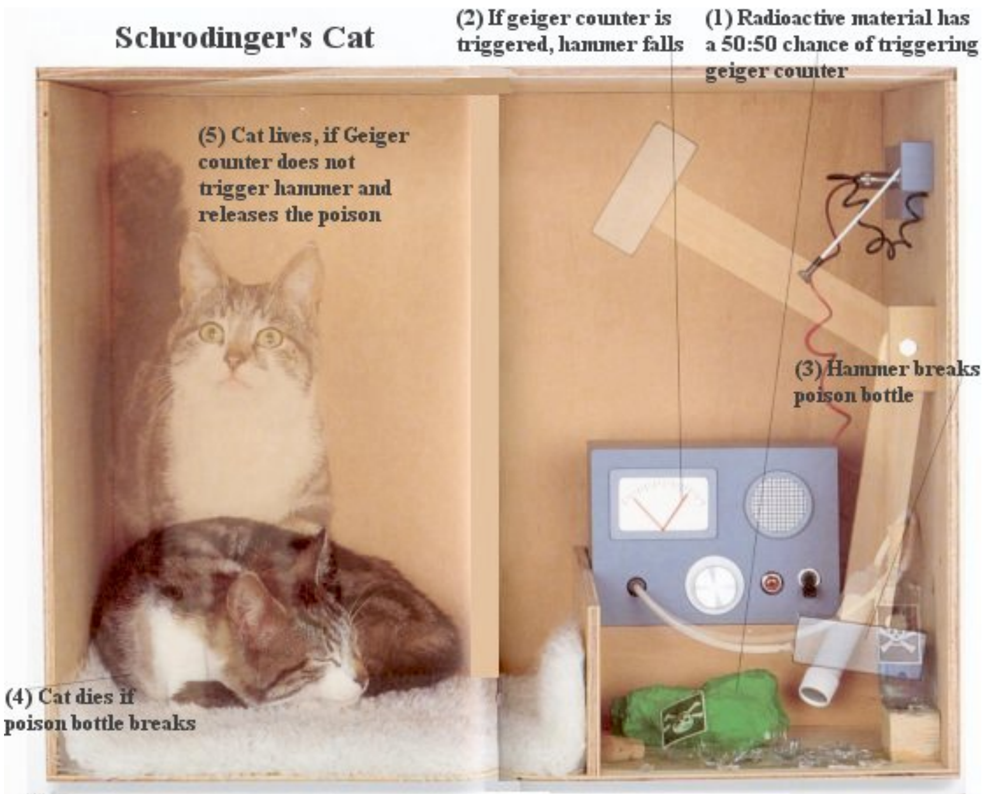
Hidden-variable theories are inconsistent with experiment.

Experiments agree with the predictions of Quantum Mechanics

Doesn't the "communication" between the entangled particles violate the laws of special relativity (e.g. the wavefunction collapse)?

Well, if you try to find a way to use this to transmit information, you will find that that is not possible. So in that sense, things are still OK.

Schrödinger's Cat



This scenario was originally developed to support the realist view of quantum mechanics, since the idea that the cat exists in a superposition of states seems absurd.

However, there are people who take that possibility seriously.

There are many different ways of thinking about this:

For eaxmple: doesn't the Geiger counter make a measurement that collapses the wavefunction? What do we mean by "observing the system"?

The Many Worlds Interpretation

There are lots of places to read about interpretations of QM, including Schrödinger's cat. And there are many "interpretations" including (as one example) the "Many Worlds Interpretation" (Everett, 1957) in which (staying with Schrödinger's cat as an example) both the dead and alive states of the cat persist after observation:

In this interpretation the observer's state becomes entangled with the cat's state and both outcomes co-exist (but there can be no communication possible between the two).

This "splitting" takes place each time a quantum mechanical process with multiple possible outcomes takes place

In this view of things, there is no "wavefunction collapse"

You can form your own opinions on this, but there are people who take this seriously.

THE END

I will post some more suggested problems in the next couple of days.

Solutions to all suggested problems (from the Modern Physics part of the course) will be posted as well, at some point prior to the exam.

I will post an announcement about office hours prior to the exam (these will probably be the Friday and Monday just before your exam)