PHY293 Lecture #5

November 3, 2017

1. Revisit special cases of Relativistic Doppler shifts

   1 Source moving directly away from observers

$$\Rightarrow f_{obs} = f_{source} \cdot \frac{\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{v}{c}} = f_{source} \cdot \frac{\sqrt{\left(1-\frac{v}{c}\right)\cancel{\left(1+\frac{v}{c}\right)}}}{\sqrt{\left(1+\frac{v}{c}\right)\cancel{\left(1+\frac{v}{c}\right)}}} = f_{source} \cdot \left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/2}$$

   - In this case $\theta = 0$ and formula reduces to
   - Observer sees a lower frequency than emitted (with a definite prediction of the speed if emission frequency is known
   - Spectral lines of Hydrogen dominate light from stars (1.42GHz)
   - Thus conclude that hydrogen lines are mostly moving away from us – consistent with this prediction (time dilation) – eg. $v = 0.9c \Rightarrow f_{obs} = 0.23 f_H = 0.33$GHz

$$f_{obs} = f_{source} \cdot \frac{\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{v}{c}\cos\theta} \qquad \Rightarrow f_{obs} = f_{source} \cdot \sqrt{1-\frac{v^2}{c^2}} = \frac{f_{source}}{\gamma}$$

   2 Source motion perpendicular to line of sight

   (a) There is no classical analog to this frequency shift. If a fire-truck passes you perpendicular to its direction of motion you hear no frequency shift in the siren.

2. Optical Atom Traps (Laser Cooling)

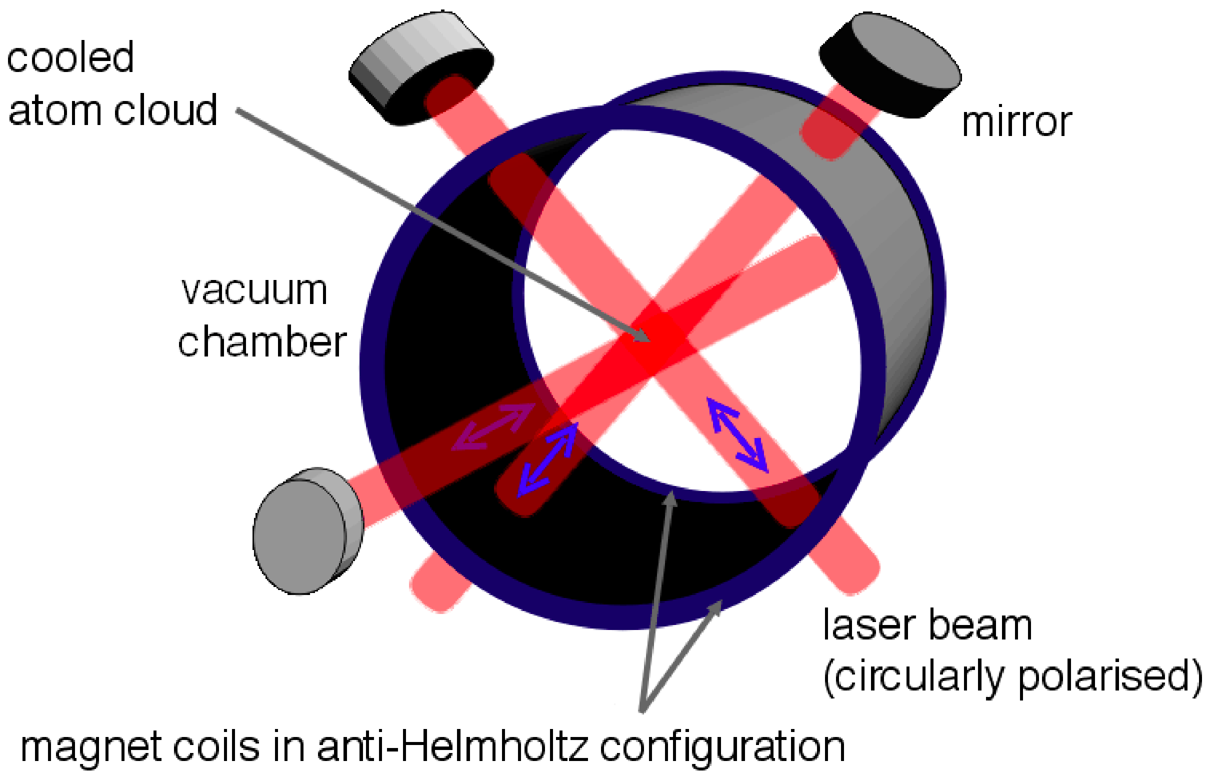
   - Additional property of light (more later in course): Each photon (particle of light) carries a momentum proportional to its wavelength
   - Momentum conserved in photon-atom interaction. When an atom that absorbs a photon gets a momentum kick in the direction of the photon
   - All atoms have set of frequencies that they can absorb (see QM section for spectral lines) eg. Hydrogen at 1.42GHz,
   - Doppler Cooling:
     - Tune a laser to just **below** a preferred absorption frequency for the atom species you want to trap
     - Only atoms moving towards laser beam (slight doppler frequency increase – blue shifted, laser coming towards the atoms) can absorb light
     - Only atoms moving out of the trap region (where you want to accumulate atoms) interact with (doppler-shifted) photons and get small kick back towards the centre of the trap
     - There was some waffling (or just plain confusion, on my part) in both classes on whether the laser should be at frequencies above or below the desired atomic absorption frequency. I hope I've got it right now.

3. Return to Twin Paradox

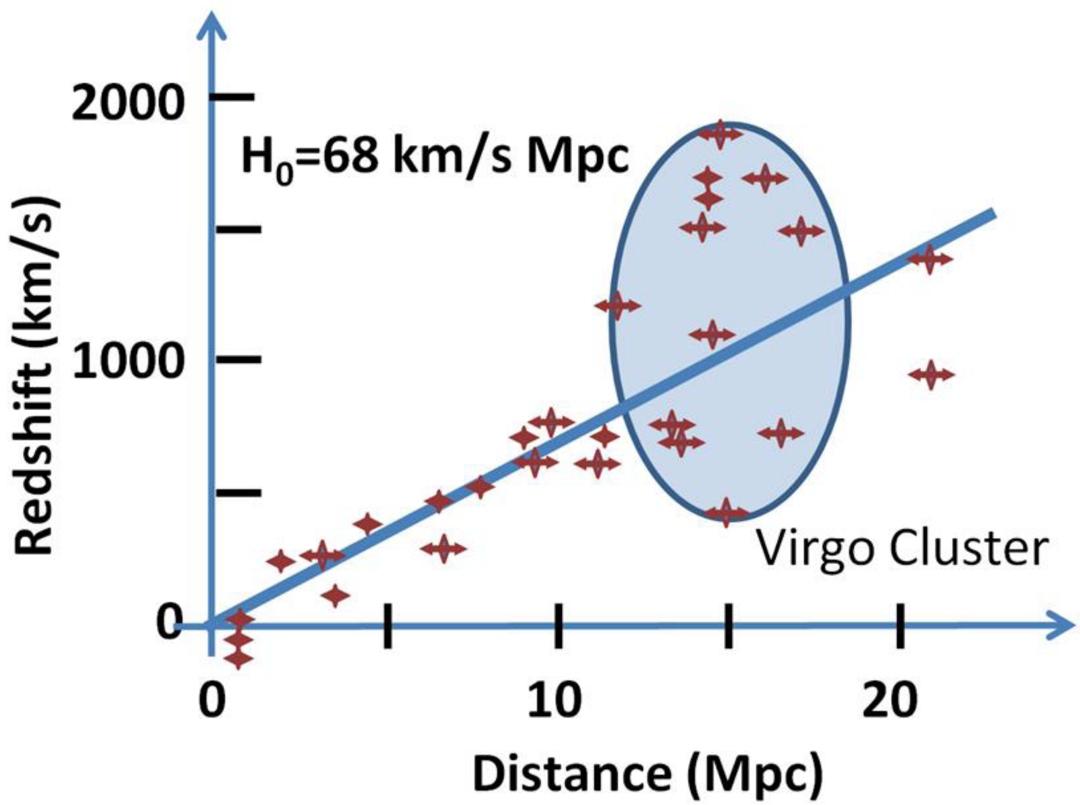
   - Recall: Anna travels from Earth to Planet 40 lyears away: at $v = 0.8c$
     - From her point of view the journey takes 30 years. At $v = 0.8c$ ($\gamma = 5/3$) the distance of the journey is length contracted: 40ly / 1.66 = 24 ly, but at $v = 0.8c$ this takes 30 years.
     - From Bob's point of view: Anna travels at 0.8c, but must cover the full 40 lyears (no length contraction – the Earth/planet distance is stationary in his frame).
     - This takes 50 years. He does observe her clock running slower (time dilation) so he agrees that she is only 30 years old when she gets there (50 yrs / 1.66 = 30).
     - Similar to cosmic ray muons. In the moving frame (Anna/muon's) the issue is length contraction, in the stationary frame (Bob/Earth Atmosphere) it's time dilation. But both observers agree on the outcome of the experiment ... Anna is 30 years old when she arrives at the distant planet.
   - To avoid an initial acceleration consider Anna born on spaceship travelling at $v = 0.8c$ at same time as Bob (OK, not twins)

- From Anna's perspective Bob's clock runs slowly (receding behind her at $v = -0.8c$).
  - Knowing that she is 30 when she arrives at planet
  - She concludes that Bob has aged less (30yrs / 1.66 = 18 years) when she arrives.
  - How can these both be true? a) Bob: Bob 50, Anna 30 b) Anna: Bob 18, Anna 30
  - Is this situation symmetric (now that we've removed the initial acceleration... maybe?)
- Rather than having Anna return to earth, we can avoid any acceleration by introducing a third participant: Carl
  - Bob and Anna already in inertial frames – constant relative velocity: $v = 0.8c$
  - Carl arrives at planet at same time as Anna, but headed towards earth at $v = 0.8c$
  - Let's assume he's arranged things to be 30 years old (same as Anna) when their ships pass, right next to planet
  - Carl will age 30 years during 'return' journey.
  - Bob will watch Carl take 50 years to return from planet to earth
  - It will be 100 years since Anna passed him (out-bound) when Carl passes him in-bound.
  - So Carl will see Bob, age 100, when they pass, but he will have seen Bob in a frame moving at $v = 0.8c$ the whole time and so will conclude that Bob has only aged 18 years during his (Carl's return journey)...
  - Carl concludes that Bob was 82 years old when he and Anna were together at the distant planet: Carl: Anna 30, Bob 82, Anna: Anna 30, Bob, 18 Bob: Anna 30, Bob 50... more and more ages
  - Let's try to come up with a way to make sense of all of this.
  - Suppose Bob sends a picture (speed of light) at age 10 (picture of him with 10 candles on birthday cake?)
    - This will take 40 years to get to Anna/Carl just as they pass at distant planet (40 light-years)
    - So it will arrive on Bob's 50th birthday (10+40).
    - This picture will show a 10yr old Bob (it has been travelling for 40 years) so it does not tell Anna/Carl anything directly about his age but, knowing SR, and their speeds relative to Bob, both Anna and Carl can calculate Bob's age as they pass at the planet.
  - Now consider how Anna measures the signal/picture and planet in her frame:
    - Anna sees the distance between Earth and Planet as 24 ly (length contracted by $\gamma = 5/3$)
    - She sees signal/picture "catching her" at $v = c$ (all observers see light at c)
    - She sees planet moving towards her at $0.8c$
    - So the separation between the signal and planet is shrinking at $1.8c$
      - NB: This is the relative separation of two different objects. No single object can move faster than c, but that is not violated here.
    - Anna will observe the signal travel 24 lyears at $1.8c$, so take a time $t = d/v = 13.3$ years
    - This is on her clock. Bob's clock runs slower by $\gamma = 5/3$ so for him the time is 40/3 / 5/3 = 8 years. So when Anna receives the picture/signal Bob is 10+8 years old. As we concluded before.
  - Carl actually sees the light signal and planet approaching him (before he and Anna pass each other)
  - The relative velocity of the signal and planet is only $c - 0.8c = 0.2c$
  - Carl sees the distance between earth and the planet at 24 lyears (contracted by the same amount as in Anna's frame)
  - Signal takes 24 lyears/$0.2c$ = 120 years to get from earth to the planet (where Carl and Anna cross-paths)
  - Bob's clock also runs slow for Carl, so Carl concludes that for Bob this time (the 120 years in Carl's frame that the signal is travelling) will take: 120 / 5/3 = 72 years.
  - So when Carl receives the signal Bob is 82 years old from Carl's perspective.
- This is at least consistent with the other way we looked at things. Pretty weird, but this is the way the Universe seems to work when things start travelling an appreciable fraction of the speed of light.

Atom Trap

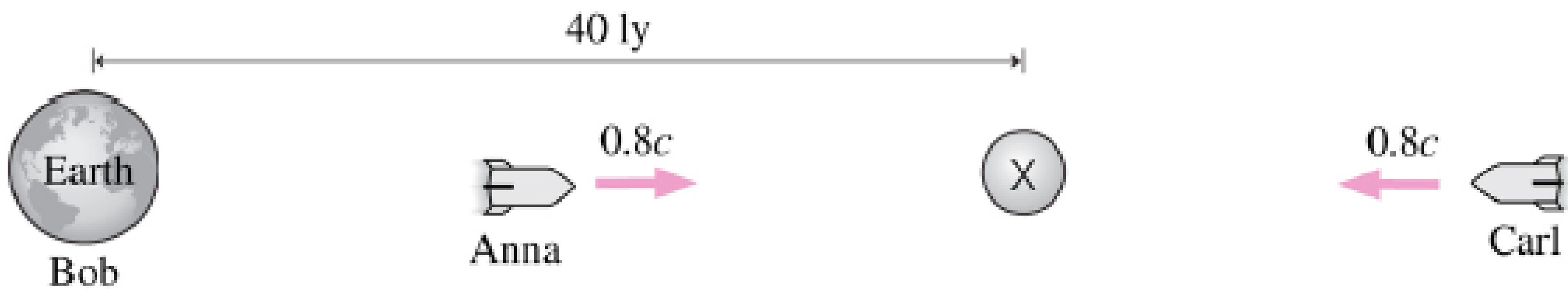

cooled
atom cloud
mirror
vacuum
chamber
laser beam
(circularly polarised)
magnet coils in anti-Helmholtz configuration

Red Shifts



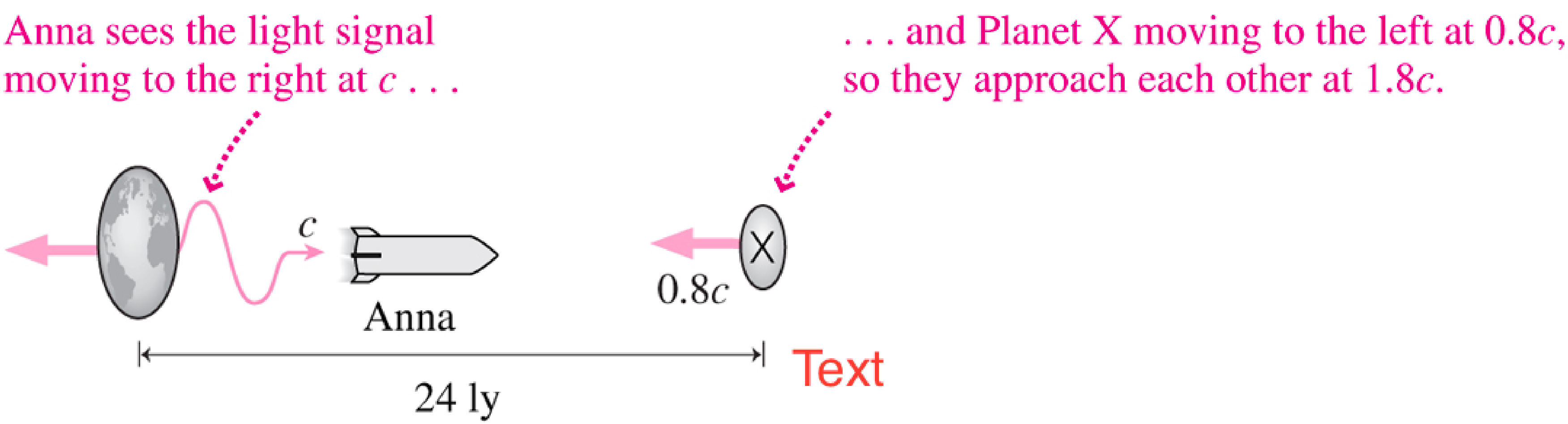
Redshift (km/s)
2000
1000
0
H_0=68 km/s Mpc
Virgo Cluster
0
10
20
Distance (Mpc)

Full Twin Paradox: Third observer



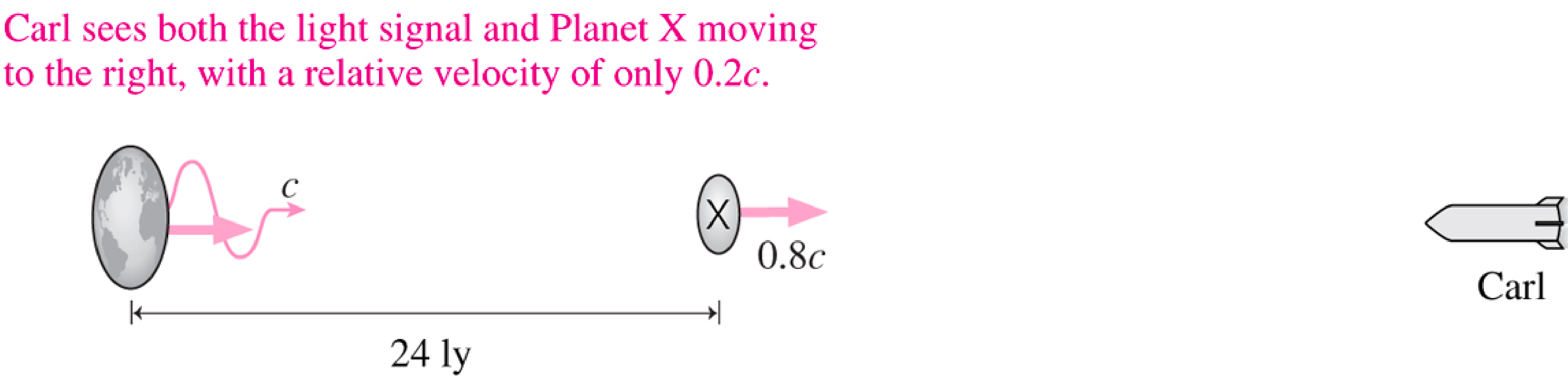
According to Bob

Anna's View



(b) According to Anna

Carl's View



(c) According to Carl