PHY293 Lecture #8

November 10, 2017

1. Applications/examples of relativistic kinematics

   - We defined the Energy-Momentum 4-vector $(E/c, p_x, p_y, p_z)$ and the associated Lorentz invariant $I = (E/c)^2 - |\vec{p}|^2 = m^2c^2$ if you sub in $E = \gamma mc^2$ and $\vec{p} = \gamma m\vec{v}$
     - If you have a single particle the invariant is just the mass-squared of the particle
     - If you have system of particles then it is the combined invariant mass of all of them
     - Since it is an invariant it will have the same numerical quantity for all observers
     - Also same value before/after any collisions or particle decays
   - Brief comment on massless particles. For $m = 0$ our formula for energy $E = mc^2/\sqrt{(1-\beta^2)}$ becomes undefined (0/0) if they travel at $\beta = 1$ (or else they have 0 energy ... which it turns out they don't).
   - Turns out not being able to use the Energy formula "as is" is OK
   - Instead return to the invariant $(E/c)^2 - |\vec{p}|^2 = m^2c^2$. For $m = 0$ this gives $E/c = |\vec{p}|$ or $E = pc$. This is how we define energy for photons/massless particles.

2. Three kinds of processes

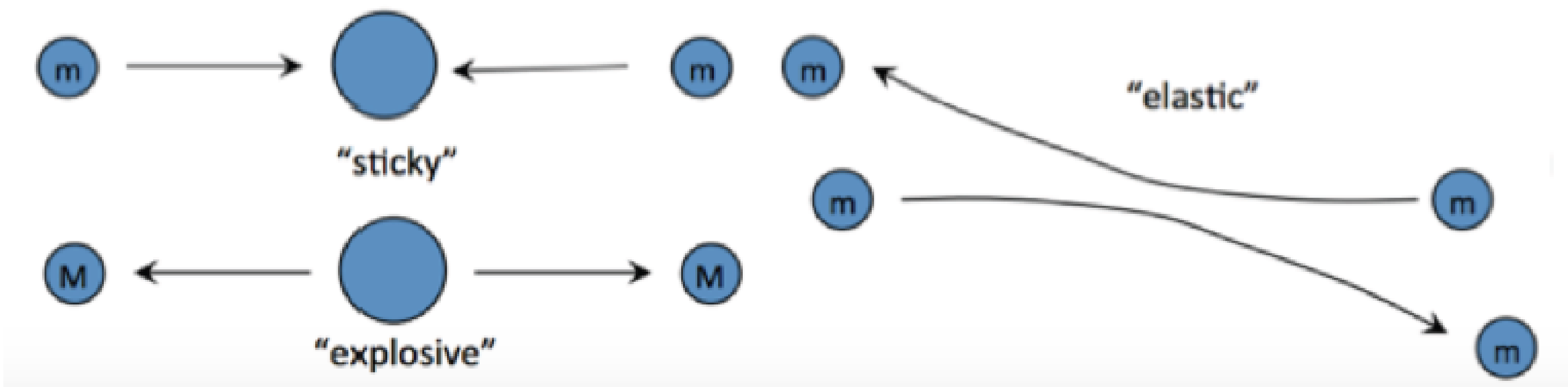
   - Sticky collisions (all particles clump together in final state), Explosive processes (single particle splits/decays into two (or more), and elastic collisions
   - Classically the mass is conserved in all such reactions, linear momentum is conserved and (at least in elastic collisions) the kinetic energy is conserved
   - In relativity
     - Energy and momentum are always conserved
     - Kinetic energy may, or may not, be conserved
     - Mass may, or may not, be conserved **but** they are either both conserved (mass/kinetic energy) or both violated
     - As we'll see in an example, if they are both violated, it happens in such a way that kinetic energy is converted to mass, or mass to kinetic energy, so that total energy is conserved. According to prescription of special relativity (ie. that Energy-momentum Invariant (a.k.a. mass^2) is conserved !)
   - Relativistic Collision
     - Consider two particles of mass m that collide, with $\beta = \pm 0.6$ and 'stick' to produce a new particle of mass M.
     - What is the mass of the final state? The linear momentum?
     - By inspection the incoming particles have the same mass, so equal/opposite linear momentum: The momentum of the initial state is 0, so conservation of momentum dictates that the final state particle will be at rest.
     - Conservation of energy says $E_1 + E_2 = E_f$, but $E_1 = E_2 = \gamma mc^2$, so $E_f = 2\gamma mc^2$
     - Since final particle is at rest ($p_f = 0$) we get $Mc^2 = 2\gamma mc^2$ or $M = 2\gamma m$
     - With $\beta = 3/5$ we get $\gamma = 1.25$ so $M = 2.5m$
     - So mass is not conserved in this collision, but neither is kinetic energy (some in initial state, none in final state).
     - But the kinetic energy of the initial state particles has been converted into mass in the final state (the extra $0.5m$)
     - Cartoon example of the kind of thing we do at the LHC ... turn some of the kinetic energy of the incoming protons into heavier objects (like Higgs bosons) to study them
   - Particle Decay
     - Now consider a single particle of mass M that decays into two particles of smaller mass m.
     - Momentum conservation requires that they be emitted in opposite directions, with the same speed (they have the same mass) so that they will have equal magnitude (but opposite sign) momentum
     - What is the velocity of the final state particles?
       * Invert the considerations from above $M = 2\gamma m \Rightarrow \gamma = M/2m$
       * This in turn gives $1 - v^2/c^2 = (2m/M)^2$ which finally yields $v = c\sqrt{1-(2m/M)^2}$
       * Note that this only makes sense for $M \geq 2m$ namely the final state particles must each be less than half of the mass of the initial state particle if they are to have a real velocity

    - So here some of the rest mass has been converted to kinetic energy. Neither individually conserved but the total energy, as defined by SR, is conserved.
  - Could also have done with the Lorentz Invariant
    - Consider the energy-momentum Lorentz invariant in the initial state:$P = (Mc, 0, 0, 0) \Rightarrow P^2 = M^2c^2$
    - In the final state $P_{final} = P_1 + P_2$ with $p_{1,2} = (\gamma mc, \pm\gamma mu, 0, 0)$
    - Lorentz invariant becomes $P^2_{final} = (2\gamma c, 0, 0, 0)^2 = 4\gamma^2 m^2 c^2$ and come to same conclusion $M = 2\gamma m$ as before
    - But with about 2 lines of algebra
  - This can be exploited to solve the case where the masses of the decay products are not equal: Pion decay
    - Here $\pi^+ \to \mu^+ + \nu_\mu$, $m_\pi = 135\mathrm{MeV}/c^2$, $m_\mu = 105\mathrm{MeV}/c^2$ and $m_\nu \approx 0$
    - In the initial state we still have $P_\pi = (m_\pi c, 0, 0, 0)$
    - In the final state we have $P_{final} = P_\mu + P_\nu = (p_\mu, 0, 0, E_\mu/c) + (-p_\mu, 0, 0, E_\nu/c)$
    - Conservation of 4-momentum requires $P_\pi = P_\mu + P_\nu$ or $P_\mu = P_\pi - P_\nu$
    - We can just square this to get various Lorentz invariants: $P^2_\mu = P^2_\pi + P^2_\nu - 2P_\pi \cdot P_\nu$
    - The muon, pion and neutrino invariants are just the masses of the respective particles (in some frame they are at rest, and all observers will agree on the size of the Invariant, so in all frames they will just be m^2c^2)
    - This gives $m^2_\mu c^2 = m^2_\pi c^2 + 0^2 - 2[m_\pi c \cdot E_v - \vec{p}_\pi \cdot \vec{p}_\nu]$
    - But $\vec{p}_\pi = 0$ (it is at rest before the decay) so the last term on RHS is 0
    - Using $E_\nu/c = |\vec{p}_\nu|$ (for a massless particle) we get: $|\vec{p}_\nu| = |\vec{p}_\mu| = \frac{m^2_\pi - m^2_\mu}{2m_\pi}c$
    - Much more complicated to do without the energy-momentum invariant. Not impossible, but longer.

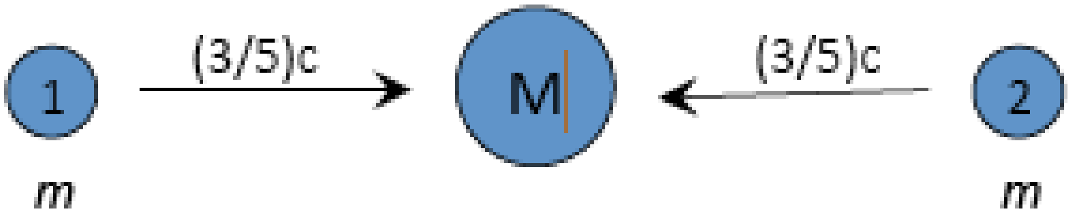
3. Intro to Quantum Mechanics

- We will try to cover the following topics in the second half of this module:
  (a) Planck suggested EM/photon energy is quantised in chunks with $E = hf$ (or $E = h\nu$)
    - Solves "Black Body radiation" problem – the other of Kelvin's 'problems with classical physics'
    - Required the idea that light energy was quantised in discrete values
  (b) Einstein went one step further and introduced photon as a particle of light
    - Did this to explain the photo-electric effect (that Kelvin missed as being a problem)
    - Suggested that each photon carries $E = h\nu$ and that Planck's quantisation of light was just the fact that light energy was broken up into individual pieces (photons).
  (c) These ideas were supported by other experiments (that we'll look at)
    - X-ray production
    - Compton scattering
  (d) Next we'll turn to a series of experiments/measurements that led to the modern atomic models
    - The Thompson model
    - The Rutherford model
    - The Bohr model – essentially the model that Quantum Mechanics predicts and that we still use today
  (e) Given that light could be both particles and waves: deBroglie suggested electrons could also be described as waves, with a wavelength related to their momentum
    - This led to diffraction and interference experiments (that had been used to understand light) also being performed on beams of electrons.
    - But particles are also a wave, what is oscillating (ie. no ether for light), what kind of wave-equation do we need to describe the physics of these waves?
  (f) This ultimately leads to Schrodinger's equation – describing the waves associated with non-relativistic particles
  (g) Aim to end up using Schrodinger's equation to solve the "particle in a box" problem. A simplified (one dimensional) version of a simple atom.

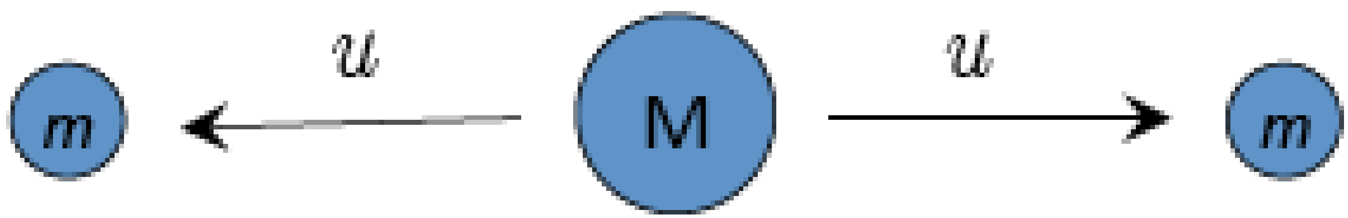
Collisions



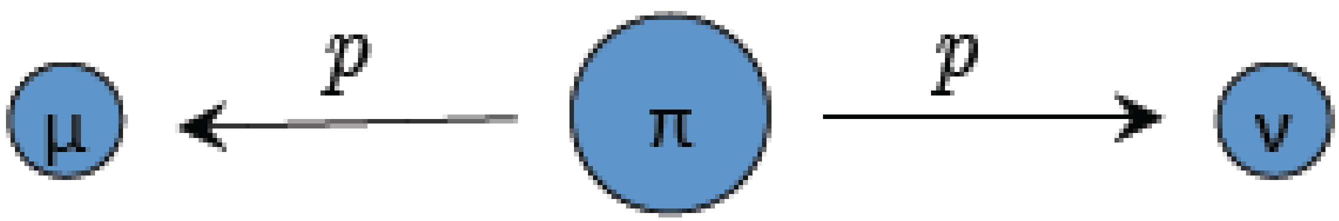
Sticky Collision



Simple Decay

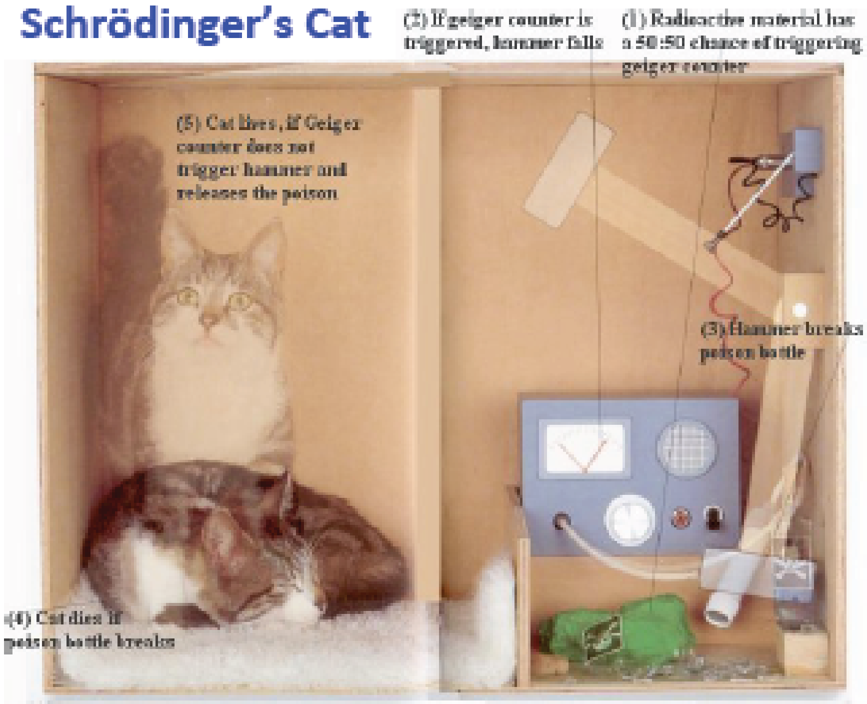


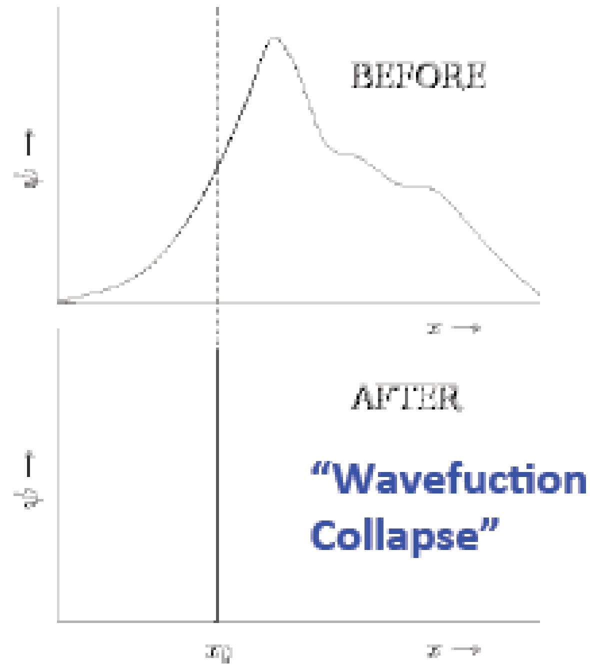
Pion Decay

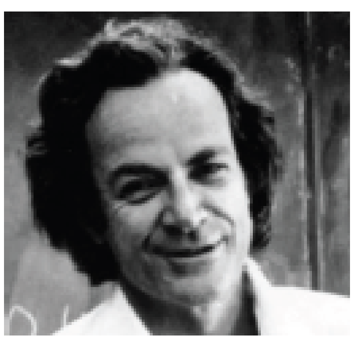


Probabilities in Quantum Mechanics

Schrödinger's Cat







"There was a time when the newspapers said that only twelve men understood the theory of relativity. I do not believe there ever was such a time. There might have been a time when only one man did, because he was the only guy who caught on, before he wrote his paper. But after people read the paper a lot of people understood the theory of relativity in some way or other, certainly more than twelve. On the other hand, I think I can safely say that nobody understands quantum mechanics."

Richard Feynman