# PHY293 Lecture #10

November 14, 2017

1. Photon Energies
   - A laser pointer (Green) emits 5mW of laser power at 532nm
   - How many photons are being emitted at that wavelength and power?
     - $E = h\nu$, so each photon has $E = hc/\lambda = 1240\text{eVnm}/532\text{nm}$ = 2.33 eV
     - Power = Energy/time so number of photons (1/s) = Power/Energy = 5mW/2.33eV
     - Need to convert MKS units to atomic units: 1 eV = $1.6 \times 10^{-19}$ J
     - So 5mW = 5 mJ/s /1.6 $\times 10^{-19}$ eV/J is 3.1 x $10^{16}$ eV/s
     - Giving finally $1.3 \times 10^{15}$ photons per second
     - Easy to see how many photons are coming out that visible light could be mistaken for a continuous phenomenon.
     - Existence of photons remained a topic of debate even in to the 1920s
2. X-ray Production
   - X-rays first discovered by Roentgen (1895) – Won first Nobel Prize in 1901
   - Classically known that an accelerating electric charge emits EM radiation
   - Produce X-rays by smashing electrons into a metal target – abrupt deceleration
   - The kinetic energy of electrons is turned in to EM energy (photons) in the keV range
   - What about the distribution of photon energies (frequencies, or wavelengths)?
     - Should the minimum wavelength (maximum frequency) depend on number/intensity of electrons?
     - The energy of the electrons?
     - No cutoff at all – This was the classical prediction
   - Experimentally, see no energy emitted at wavelengths below some cutoff
     - Cutoff depends on kinetic energy of electrons (voltage on X-ray tube)
     - Textbook uses 25 keV electrons, yielding a 0.05nm cutoff
     - Another example (see figures) shows 0.035nm for 35 kV tube
     - Also some prominent peaks – due to deep shell electrons, and depend on material
   - Quantum solution: EM energy is quantised in photons then a maximum energy – the energy of electron in X-ray tube (35kV)
   - This follows from the 'fact' that each electron can produce no fewer than one, complete, photon (could, in principle, produce 2 or more, but they would be lower energy, lower frequency and hence longer wavelength).
     - Can predict this cutoff from the formulae we've already seen

$$E_{max} = hc/\lambda_{min} \Rightarrow \lambda_{min} = hc/E_{max} = 1240\text{eVnm}/35 \times 10^3\text{eV} = 0.035\text{nm}$$

   - Absorption of X-rays depends on density of material (denser objects appear 'whiter' – fewer X-rays pass through to be recorded)
3. Compton Scattering
   - This is the study of light, scattering off electrons
     - This is typically how we study atomic and subatomic matter
     - Use some kind of probe and 'scatter it' off of target to understand target structure
     - Compton was investigating how light (and other kinds of EM radiation) was scattered from carbon targets
     - Classically, expected that electrons would oscillate in EM field (it was thought to be a wave, classically)
     - As they oscillated, they would emit EM radiation in all directions (re-scattering the incoming light)
     - Since oscillating at frequency of incoming radiation, the re-scattered radiation would be at that same frequency.
   - Compton (1923) used X-rays scattered off carbon to understand what happened
     - The electrons are very loosely (on the scale of the X-ray energy) bound to the carbon atoms

- Atomic binding energy is O(100 eV)
- Compton used Molybdenum X-rays ($\lambda = 0.0708$ nm)
- Found that some radiation was scattered **backwards** at a different wavelength
- Found a proportionality between the scattering angle and the wavelength of light emitted
- At the same time the photon energies are large enough to produce relativistic electrons
  - $\lambda = 0.0708\text{nm} \Rightarrow E = hc/\lambda = 1240\text{eVnm}/0.0708\text{nm} = 17.5$ keV
  - This is a few $\%$ of the mass of an electron (511 keV)

- Compton explained this as a 'billiard ball' collision
  - Between an incoming photon with $E_\gamma = p_\gamma c = h\nu = hc/\lambda \rightarrow p_\gamma = h/\lambda$
  - An electron initially at rest (orbiting carbon atom with momentum that corresponds to $E_{kinetic} \approx 100$ eV, small compared to X-ray energy coming in)
  - After collision photon emerges at some angle $\theta$, with some different energy: $E'_\gamma = p'_\gamma c = hc/\lambda' \rightarrow p'_\gamma = h/\lambda'$
  - The electron gets a kick emerging with some momentum, along direction $\phi$: $E'_e = \gamma m_e c^2 \qquad \vec{p}_e = \gamma m_e \vec{u}_e$
  - Separately conserve energy and momentum. Momentum conserved both parallel to direction of initial photon and perpendicular to that direction (in the final state).
  - $x$ direction: $\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + \gamma m_e u \cos\phi$
  - $y$ direction: $0 = \frac{h}{\lambda'}\sin\theta - \gamma m_e u \sin\phi$
  - Energy: $\frac{hc}{\lambda} + m_e c^2 = \frac{hc}{\lambda'} + \gamma m_e c^2$
  - Can solve (problem 3.38 suggests an approach to solving this) to get:

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

  - The fact that this requires the relativistic form of the electron energy and momentum (not to mention the quantised energy/momentum of the photon) is further evidence that SR is 'right' and that light behaves as a particle.
  - Wavelength shift is independent of the wavelength of the incident EM radiation,

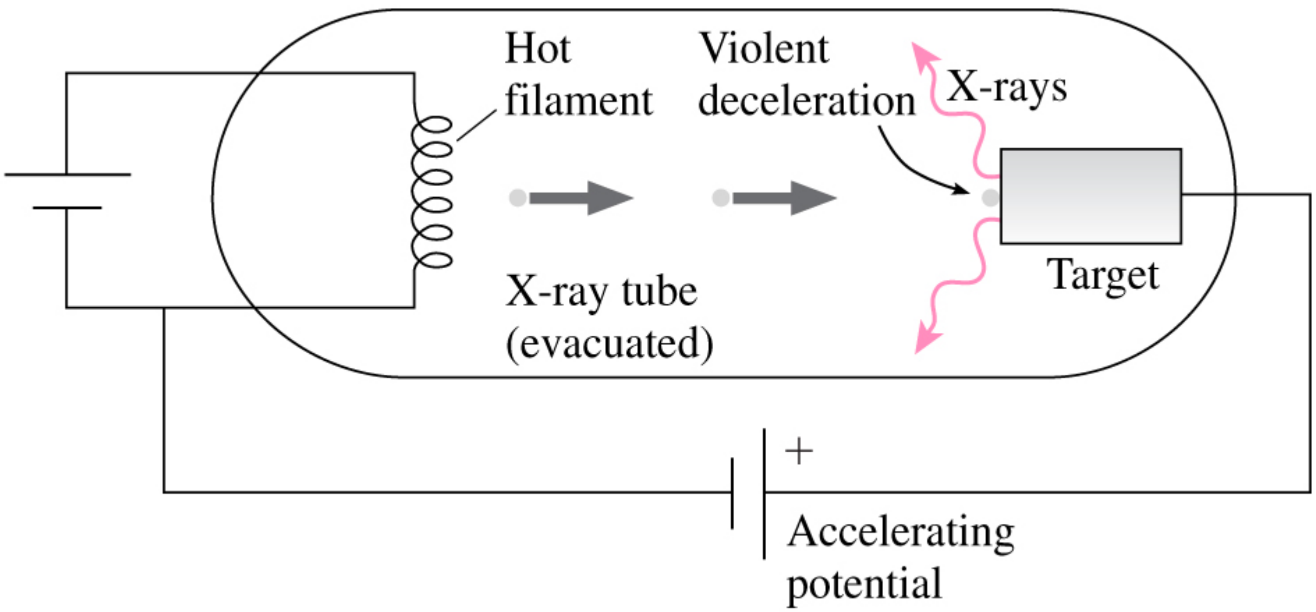
- Get a feel for 'size' of this effect at various wavelengths
  - Consider the wavelength shift $\lambda' - \lambda = \frac{h}{m_e c}O(1) \approx \frac{hc}{m_e c^2}$
  - This is easy to compute: $\frac{1240\text{eVnm}}{511\times10^3\text{eV}} = 0.0024\text{nm}$
  - Depending on the angle of scattering this could be a factor of 2 larger, or much smaller (all the way down to 0 shift for photons emitted in direction of incident photon ($\theta = 0$)
  - But it's a pretty small shift, and will be hard to 'observe' for visible light (ie. 500nm $\rightarrow$ 500.002 nm)
- Use incident EM radiation with $\lambda \approx 0.02$nm (or less) so relative shift is sizeable (10% of incident wavelength) – X-rays
- For 'long wavelengths' the light scattered in all directions will be about the same as incident wavelength
  - If we start with 500 nm visible light then we'll see
  - 500.002 nm at $\pm 90^o$, 500.004 nm at $\theta = 180^o$, and 500 nm at $\theta = 0$
  - Another example of "Correspondence principle":
    - If we start with long wavelength light, then the scattered photons go in all directions with $\approx$ the same wavelength
  - This was the classical expectation for EM waves scattering off atoms.
  - The quantum prediction merges smoothly into the classical prediction for larger wavelengths (ie. the classical regime)
- To summarise: Compton's measurements and the QM interpretation assume that the interaction of photons and electrons are just those of two individual particles, following the laws of (relativistic) momentum and energy conservation
- With the added assumption that there exists a photon with energy $E = h\nu$ and $p = h/\lambda$
- I strongly recommend Example 3.3 in the textbook.
  - The mechanical conservation laws should be familiar to you.
  - But details of what to put in for the photon energy, momentum (and even the relativistic electron energy) are new. You should practice these.

# Planck Quote

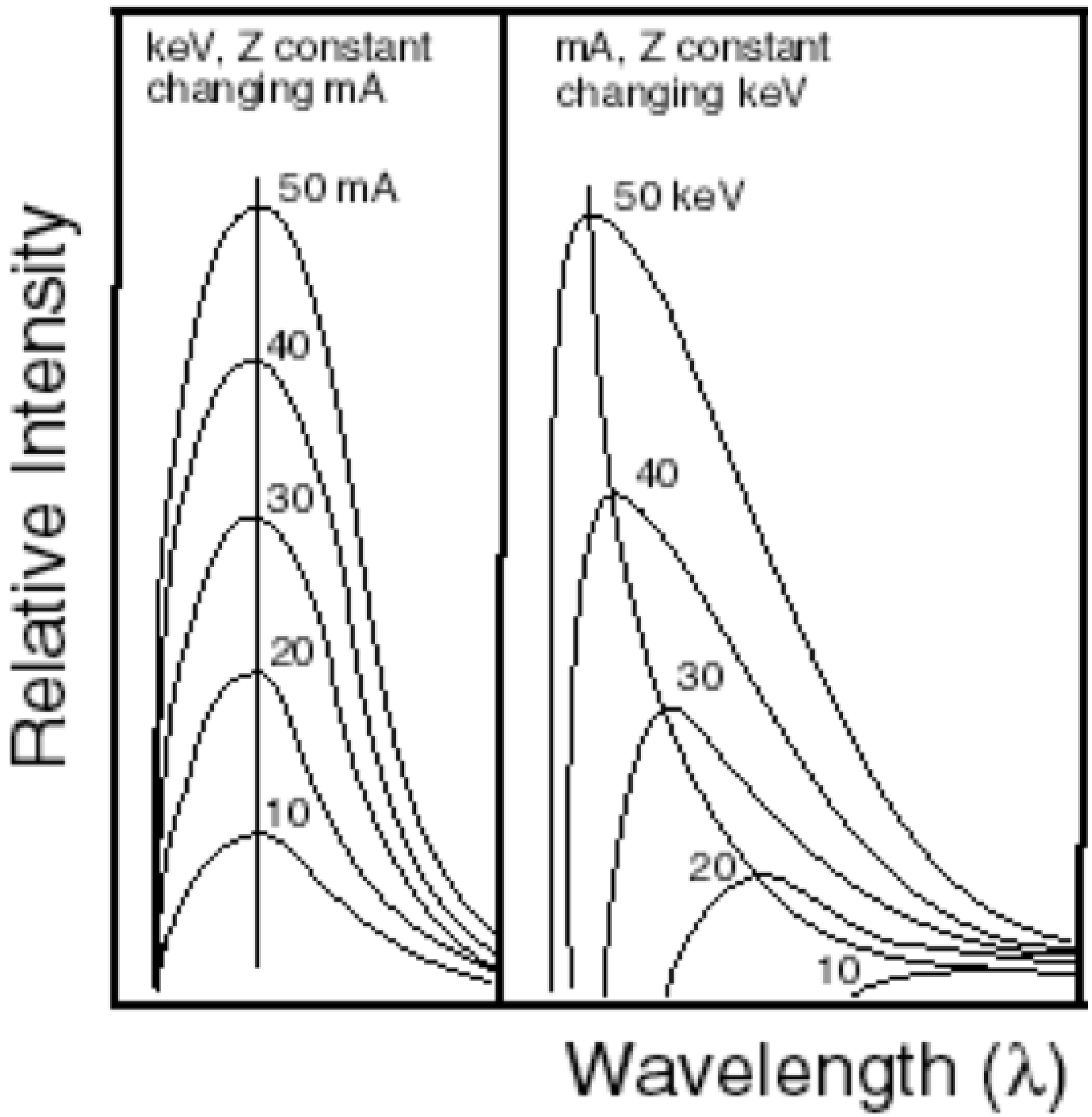
"In sum, one can say that there is hardly one among the great problems in which modern physics is so rich to which Einstein has not made a remarkable contribution. That he may sometimes have missed the target in his speculations, as, for example, in his hypothesis of light-quanta, cannot really be held too much against him, for it is not possible to introduce really new ideas even in the most exact sciences without sometimes taking a risk. "

*Max Planck*

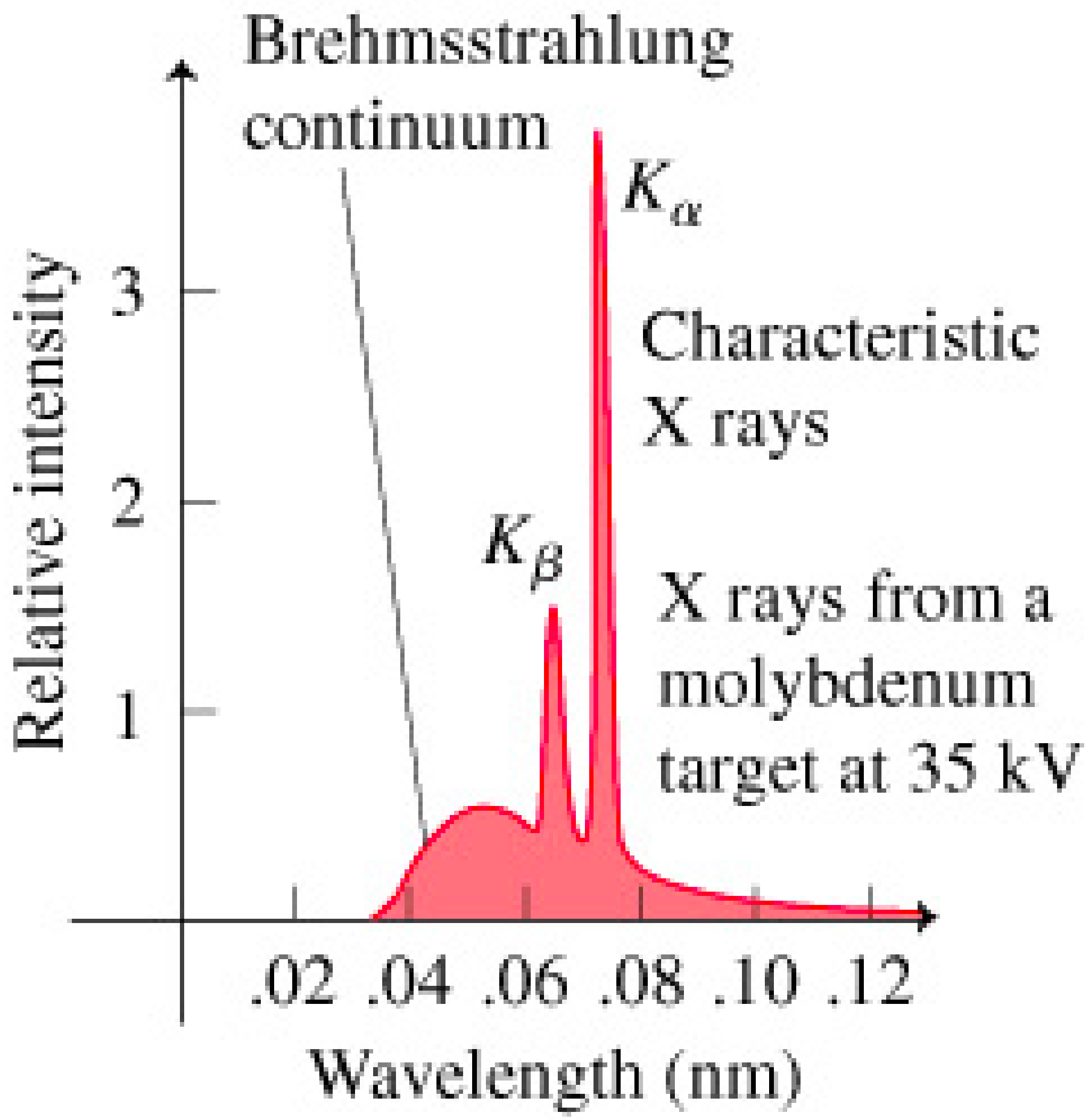
# X-Ray Tube


Hot
filament
Violent
deceleration
X-rays
Target
X-ray tube
(evacuated)
+
Accelerating
potential

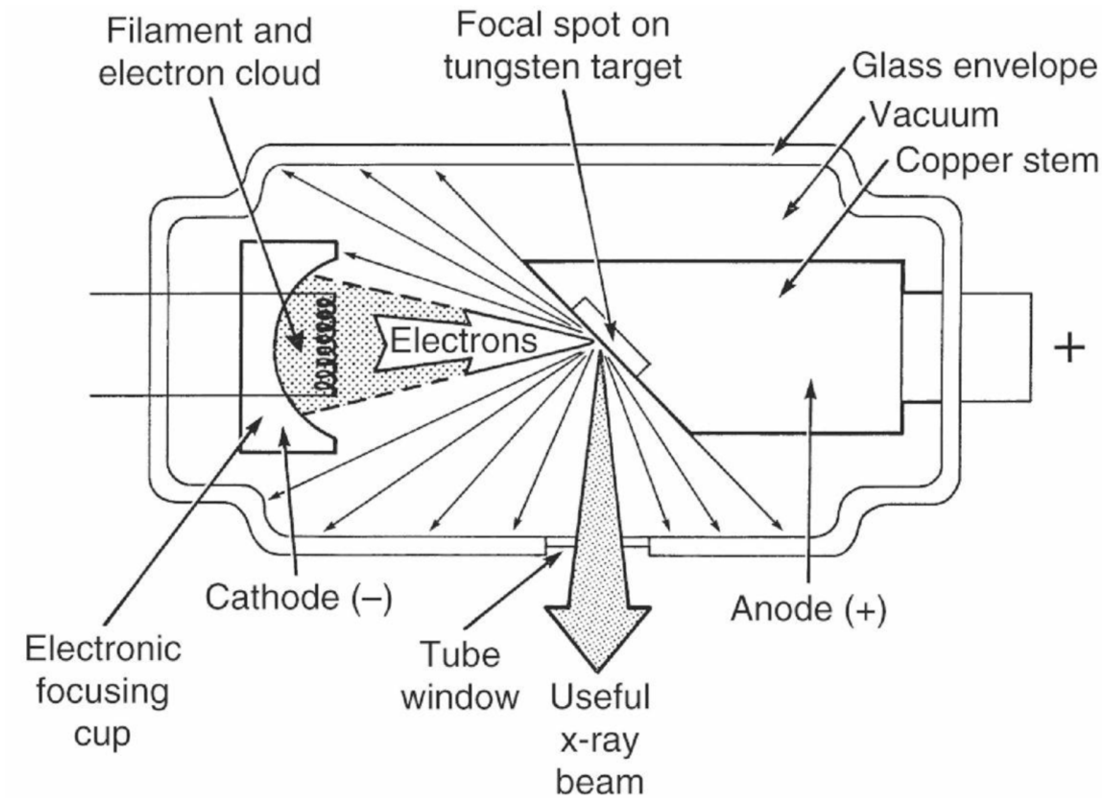
# Wavelength Distribution of X-rays



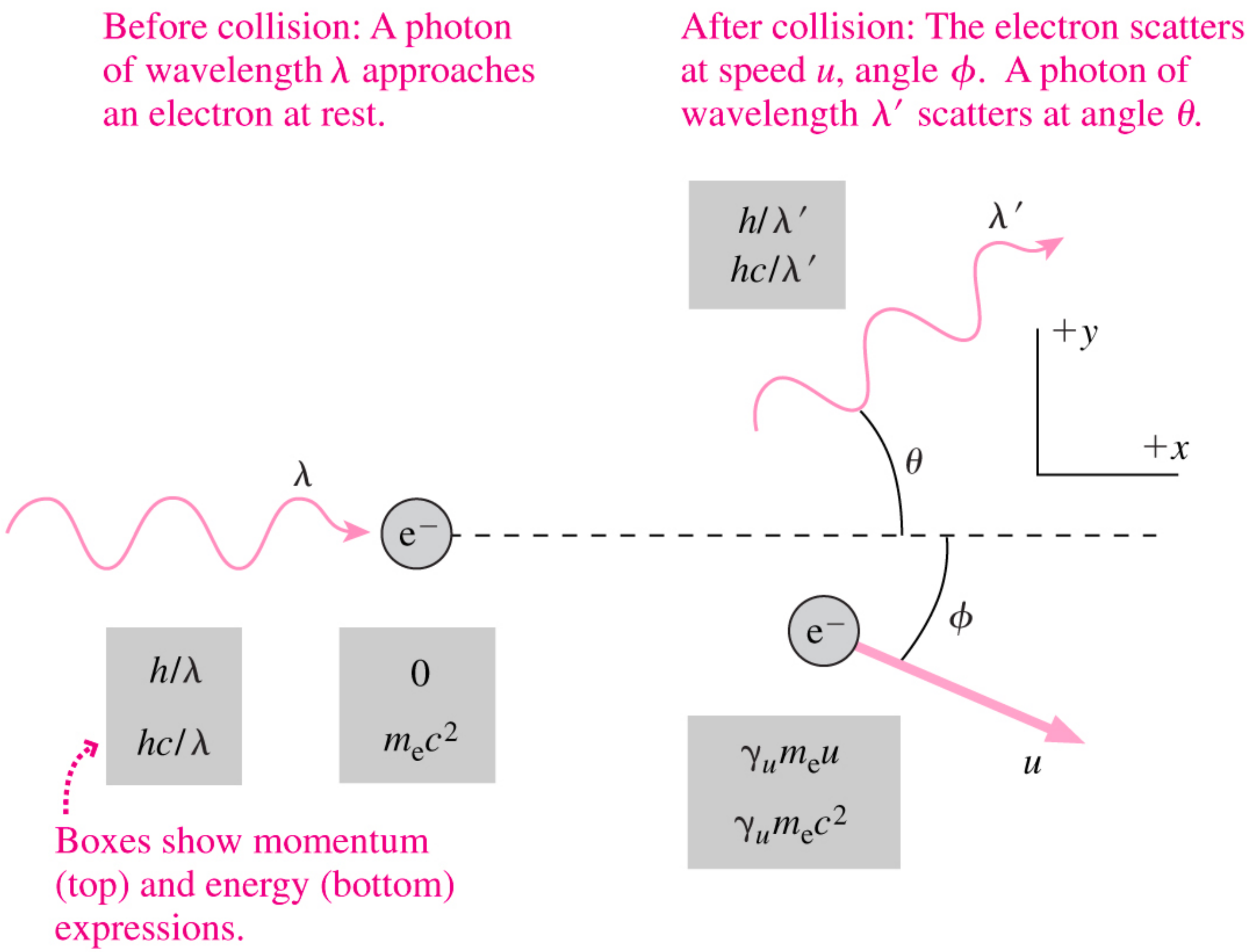
# 35 kV – Molybdenum target



# Dental x-ray tube




# Photon - Electron Scattering



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# PHYSICAL REVIEW

## A QUANTUM THEORY OF THE SCATTERING OF X-RAYS BY LIGHT ELEMENTS

BY ARTHUR H. COMPTON

ABSTRACT

**A quantum theory of the scattering of X-rays and γ-rays by light elements.**

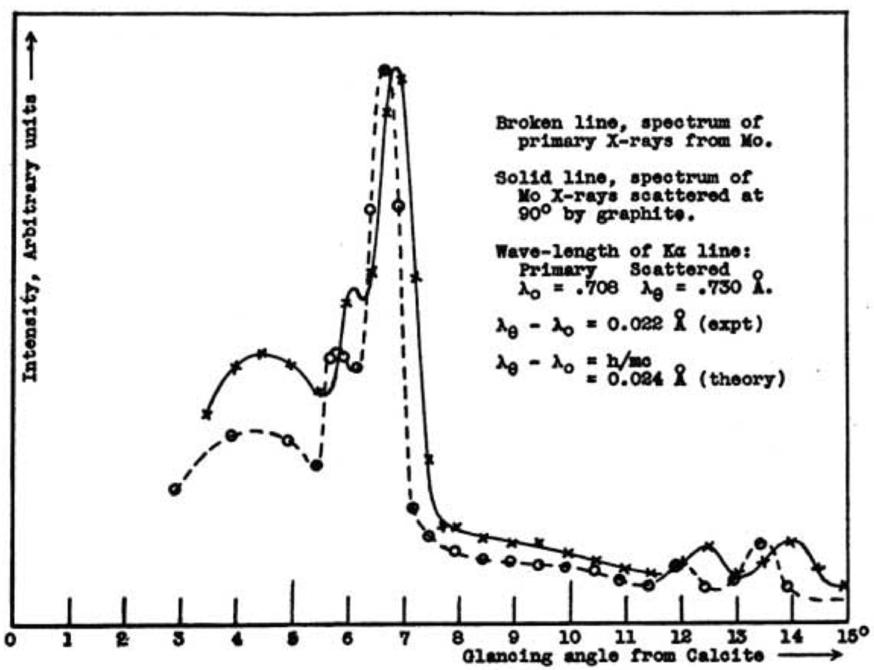
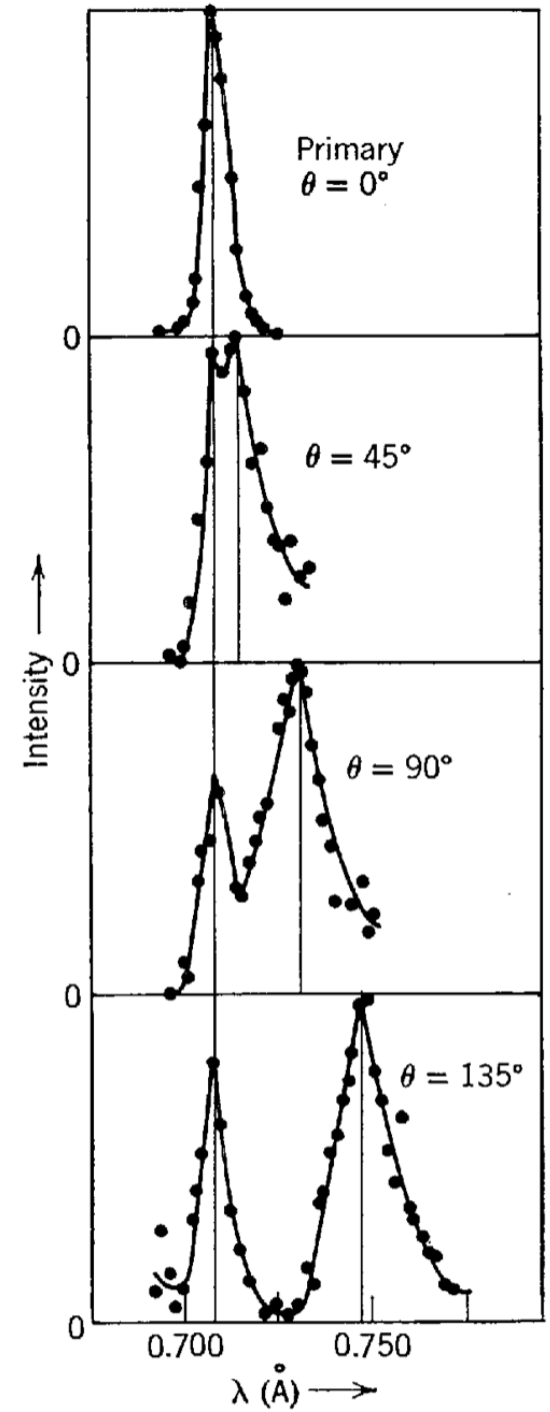



Fig. 4. Spectrum of molybdenum X-rays scattered by graphite, compared with the spectrum of the primary X-rays, showing an increase in wave-length on scattering.

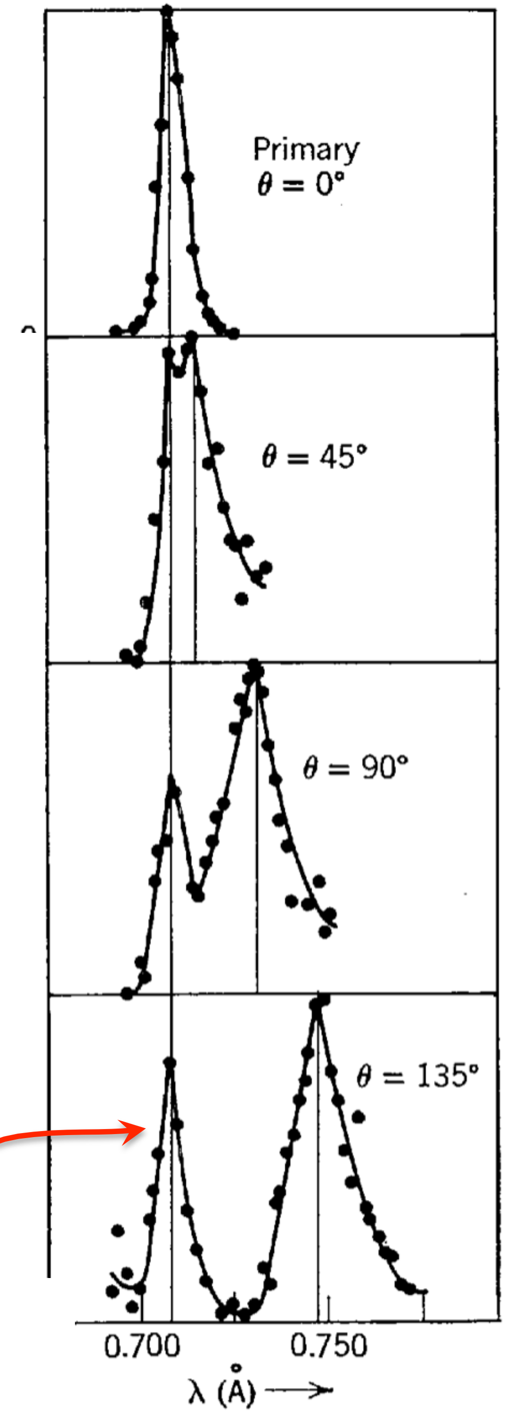


Spectrum scattered at θ = 0°

Spectrum scattered at θ = 45°

Spectrum scattered at θ = 90°

Spectrum scattered at θ = 135°




What about the peaks at $\lambda_{incident}$ for all angles?

[see problem 3.33]

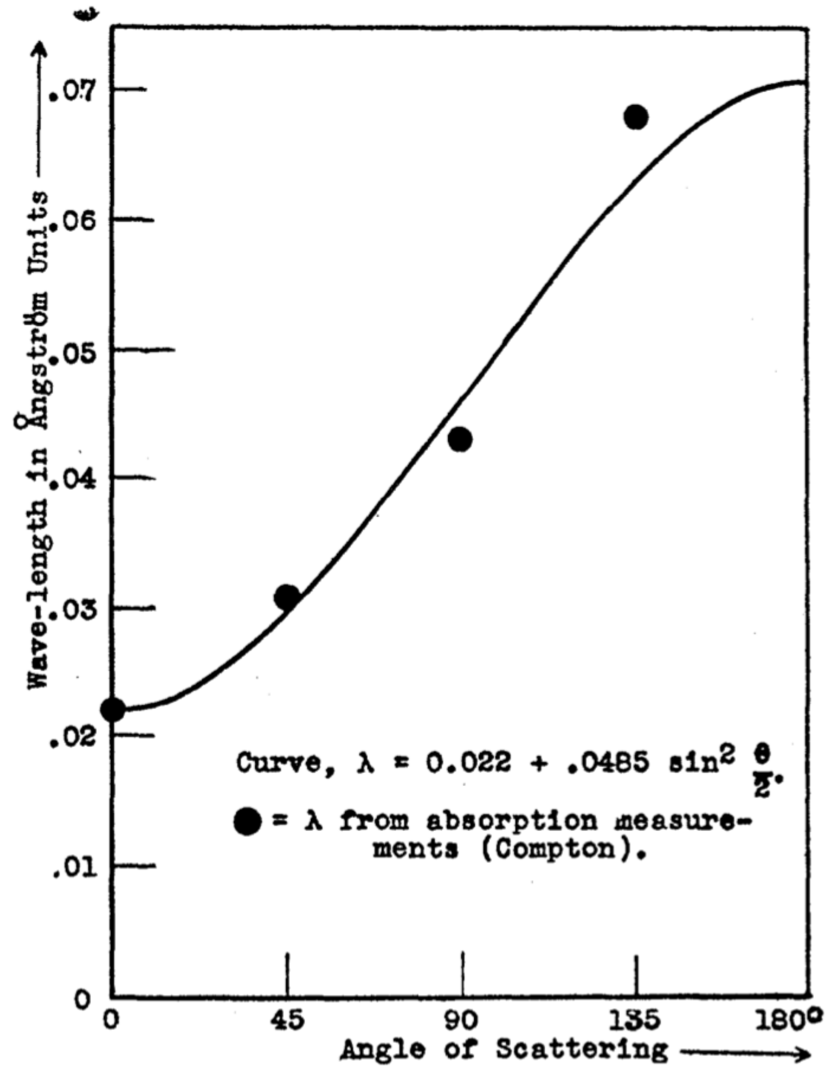



Fig. 5. The wave-length of scattered $\gamma$-rays at different angles with the primary beam, showing an increase at large angles

Compton won the 1927 Nobel prize for this work