PHY293 Lecture #11

November 17, 2017

1. Compton Scattering (reprise)
   - Initially Compton only measured the wavelength of incoming/outgoing X-rays
   - Later, Compton (and others) measured the scattered electron, at predicted angle and momentum
     - Further cementing the 'billiard ball' interpretation of photon-electron interactions
   - Novel physics in Compton scattering:
     - Quantum-mechanical prediction that the photon a) exists and b) behaves as a particle with $p = h/\lambda$.
   - Look at example 3.3 in the textbook. All kinematic considerations in solving a problem like that should be familiar. The new thing is using photons with definite energy and momentum and using relativistic expressions for the electron energy and momentum.
2. Inelastic Scattering
   - So far have just discussed elastic scattering of photons
   - Compton scattering only involved the exchange of kinetic energy from the photon to the electron
   - There are many examples where there is conversion from mass to kinetic energy, or vice-versa. These are **inelastic** collisions
   - One such example is the interaction:
   $$\text{photon} + \alpha \rightarrow d + d$$
   - An alpha (two protons and two neutrons, A Helium nucleus) particle absorbs a photon and disintegrates to two deuterons
     - Each deuteron is a bound-state of one proton and one neutron.
   - The initial state consists of a photon with wavelength λ and an α particle (^4He nucleus) at rest
   - Final state has two deuterons, with their rest energy and momentum (kinetic energy)
   - Energy must be conserved:
   $$\frac{hc}{\lambda} + m_\alpha c^2 = 2\gamma_{(v=0.6c)} m_d c^2$$
   - The momenta of the deuterons must sum to 0, perpendicular to the direction of arrival of the photon (no momentum in this direction before photon and alpha collided)
   $$0 = \gamma_{(0.6)} m_d v_d \sin\theta_1 - \gamma_{(0.6)} m_d v_d \sin\theta_2 \Rightarrow \theta_1 = \theta_2 \equiv \theta$$
   - Momentum along the direction of the photon must be conserved before/after the collision
   $$\frac{h}{\lambda} = 2\gamma_{(0.6)} m_d v_d = 2\gamma_{(0.6)} m_d \cdot 0.6c \cdot \cos\theta$$
   - The textbook solves for θ in example 3.4
   - We can at least work out the energy of the photon necessary to produce two deuterons moving this fast:
   $$\frac{hc}{\lambda} + m_\alpha c^2 = 2\gamma_{(0.6)} m_d c^2 \quad \text{using} \quad m_\alpha = 4.00151u; \quad m_d = 2.01355u; \quad 1u = 1.66 \times 10^{-27}\ \text{kg}$$
   - $1u$ (unified atomic mass unit) is roughly the mass of one free neutron or proton.
     - Precisely it is defined as 1/12th of the mass of a ^{12}C atom (which has 6 neutrons, 6 neutrons and 6 electrons)
   - The fact that the α has a mass lower than 2 free protons plus two free neutrons (proton $1.008u$, neutron $1.009u$) is a reflection of the extra mass energy that binds the protons and neutrons together in to a Helium nucleus.
   - We can re-arrange this to get:
   $$\frac{hc}{\lambda} = c^2(2m_d \cdot 1.25 - m_\alpha) = c^2(1.0342u) = 1.54 \times 10^{-10}\text{J} \approx 1\ \text{GeV}$$
   - To get the last step recall that 1 eV $= 1.6 \times 10^{-19}$ J

- Could also ask the question: What is the minimum photon energy just to split the alpha particle into two deuterons, at rest.
- Where does a 1 GeV photon fall on the electromagnetic spectrum?
  - Has a wavelength of $\lambda = \frac{hc}{E} = \frac{1240\text{eV nm}}{10^9 eV} = 1.2 \times 10^-4$ nm
  - At these energies/wavelengths it makes most sense to just talk about the energy of the photon. Essentially all interactions at this end of the spectrum will be particle-light (photons) not wave-like light.

3. Fusion as another inelastic process

- The inverse reaction is something like: $p + d \rightarrow^3 He + \text{photon}$
- This is an important contribution to solar burning – keeps us warm and alive
- Here we start with a proton and deuteron in the initial state.
- They actually bind together via the strong force – happens much quicker (ie. at much higher rates) than most other reactions in the sun
- This is another example of what photons can 'do'. Here the EM radiation is not scattered in the process. There is **no photon** in the initial state. The binding of the proton and deuteron into a ^3He nucleus releases energy in the form of a photon.

4. Electron-Positron Pair Production

- Pair production is another example of an inelastic process involving a photon
  - An electron and positron is the lightest pair of particles that can be produced in this way
  - But can produce any particle/anti-particle pair in this way ($\mu^+\mu^-$ or even proton and anti-proton)
- First question we can ask is "What is the minimum energy of photon that is needed to create a 'pair' like this?"
- Must correspond to the rest mass of the electron + positron (they are equal)

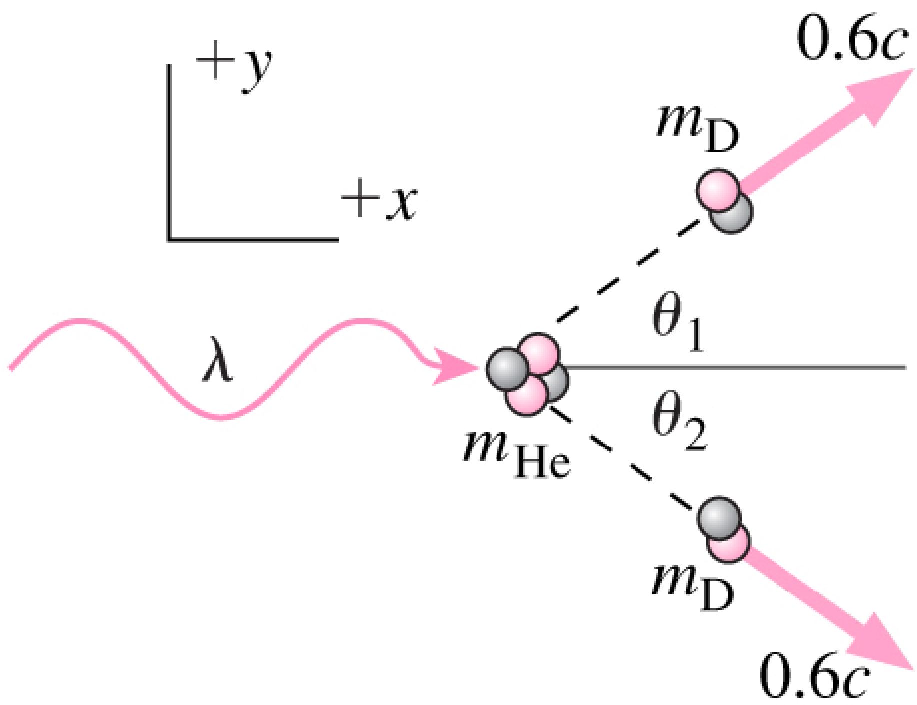
$$E_{photon} = 2m_e c^2 = 2 \times 511 \text{ keV} = 1.02 \text{ MeV}$$

- If the photon has more energy, then the electron and positron share the remainder as kinetic energy (ie. they get some momentum and are not just produced at rest).
- Note: This actually cannot happen in "free space"
  - Consider conservation of momentum. What is momentum of electrons right at threshold (1.02 GeV)? What does this imply for the momentum of the initial photon?
  - Even above threshold consider a frame where the electron and positron are emitted back-to-back (still with equal/opposite momenta). What is the overall momentum of the system in that frame?
  - Recall that momentum must be conserved in that frame too $\Rightarrow$ initial photon must have 0 momentum (be at rest). Not possible $E = pc \rightarrow$ only a photon with 0 energy has $p = c$
  - Process only proceeds in the presence of matter, where the pair production occurs in presence of a strong EM field of the nucleus
  - Text calculates the actual momentum absorbed by the nucleus (small)

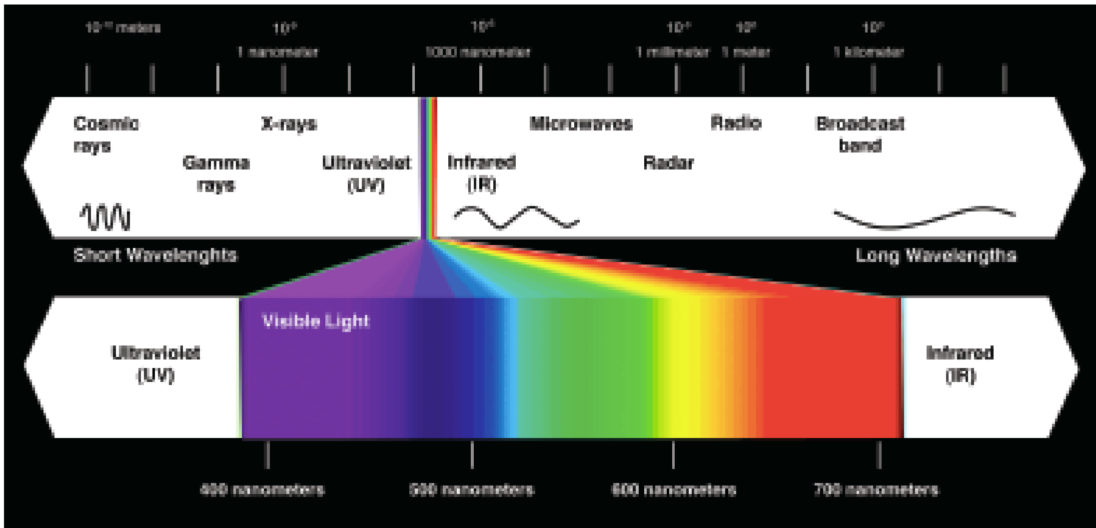
5. Photon Interactions with Matter

- Can quantify the number of interactions of photons (or any other particle) passing through matter by: $N_{int} = \sigma_{int} \cdot I$
- σ_{int} is known as the scattering cross-section and is a measure of the probability that photons will interact
- As we'll see, quantum mechanics is ideally suited to compute this kind of probability for us.
- Cross-sections for different processes can be added: $\sigma_{tot} = \sigma_{elastic} + \sigma_{photoelec.} + \sigma_{pair} + Z\sigma_{Compton}$
- In different photon energy ranges, different processes are more or less important
- Can actually measure these effects in advanced PHY labs (part of EngSci physics option - PHY427
  - Use different materials of different thicknesses to see how they attenuate beam of high energy (^{137}Cs) photons.
  - Advanced physics lab has absorbers of C, Al, Cu, Sn, Pb, Bi
  - Photons at 662 keV are ideally placed to distinguish between Compton scattering and other processes

Alpha Disintegration



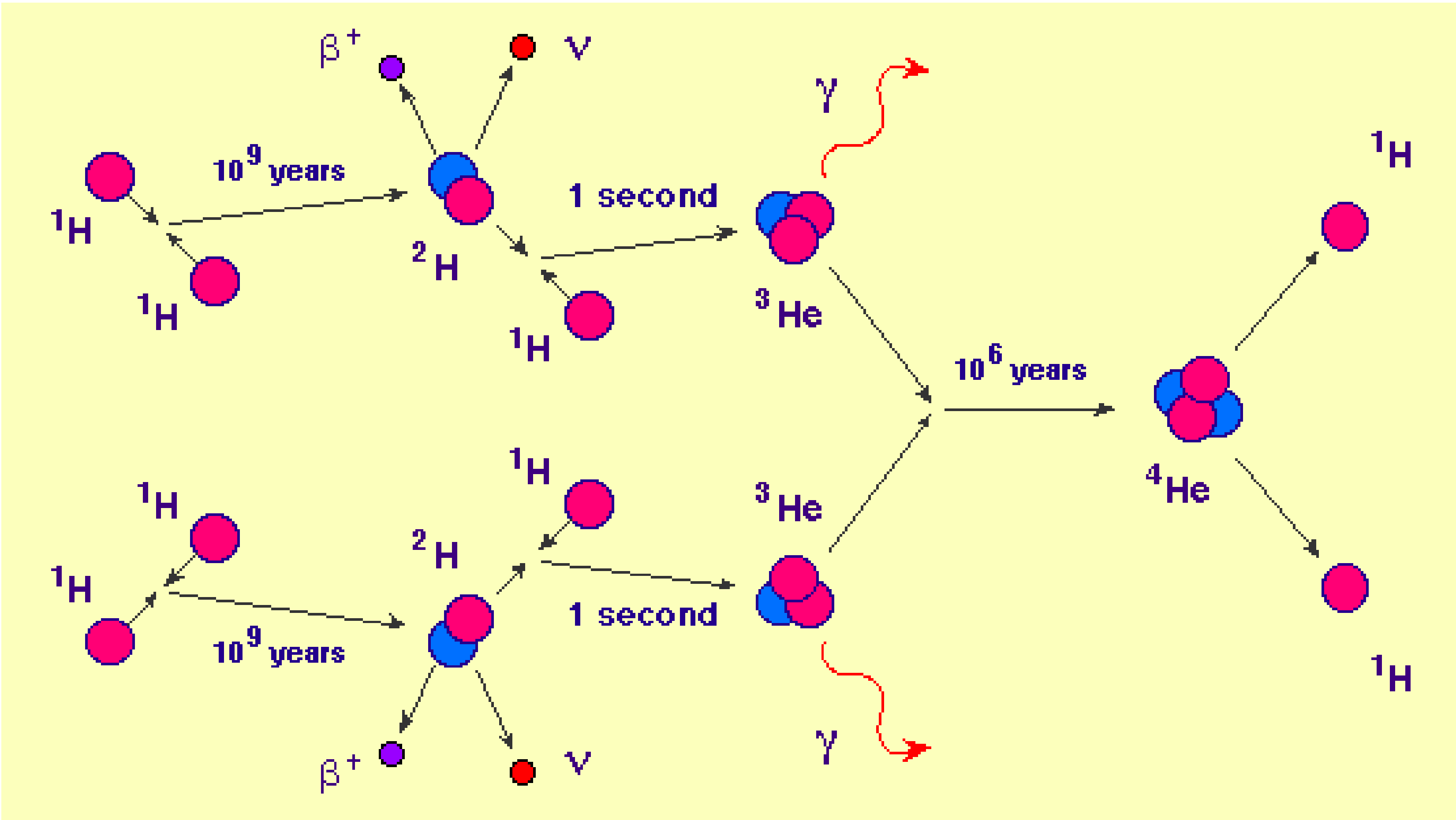
Gamma Ray Energy



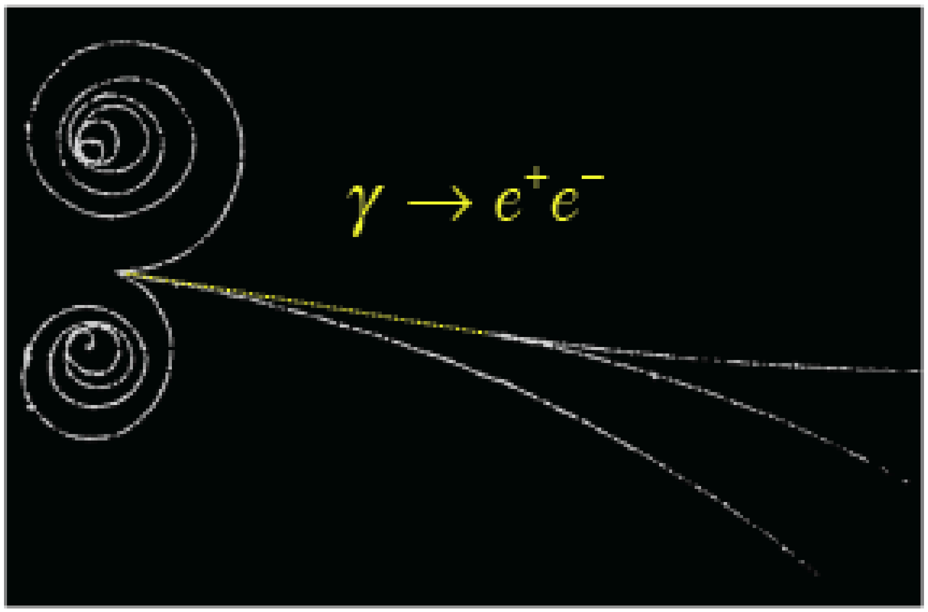
See lecture 7 for full table

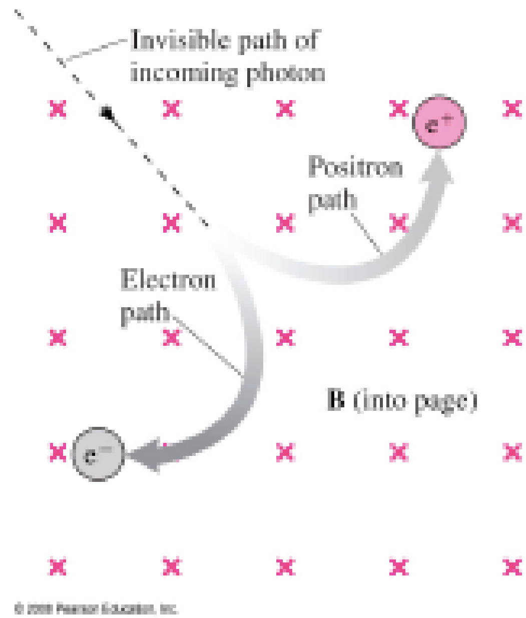
Spectrum			Energy per Photon (eV)
			10^{-9}
			10^{-7}
			10^{-6}
			10^{-5}
Infrared	10^{-5}	10^{13}	10^{-2}
Visible Light	10^{-7}	10^{15}	1
Ultraviolet	10^{-8}	10^{16}	10^{1}
X-Rays	10^{-10}	10^{18}	10^{3}
Gamma Rays	10^{-13}	10^{21}	$\geq 10^{6}$

Solar Processes

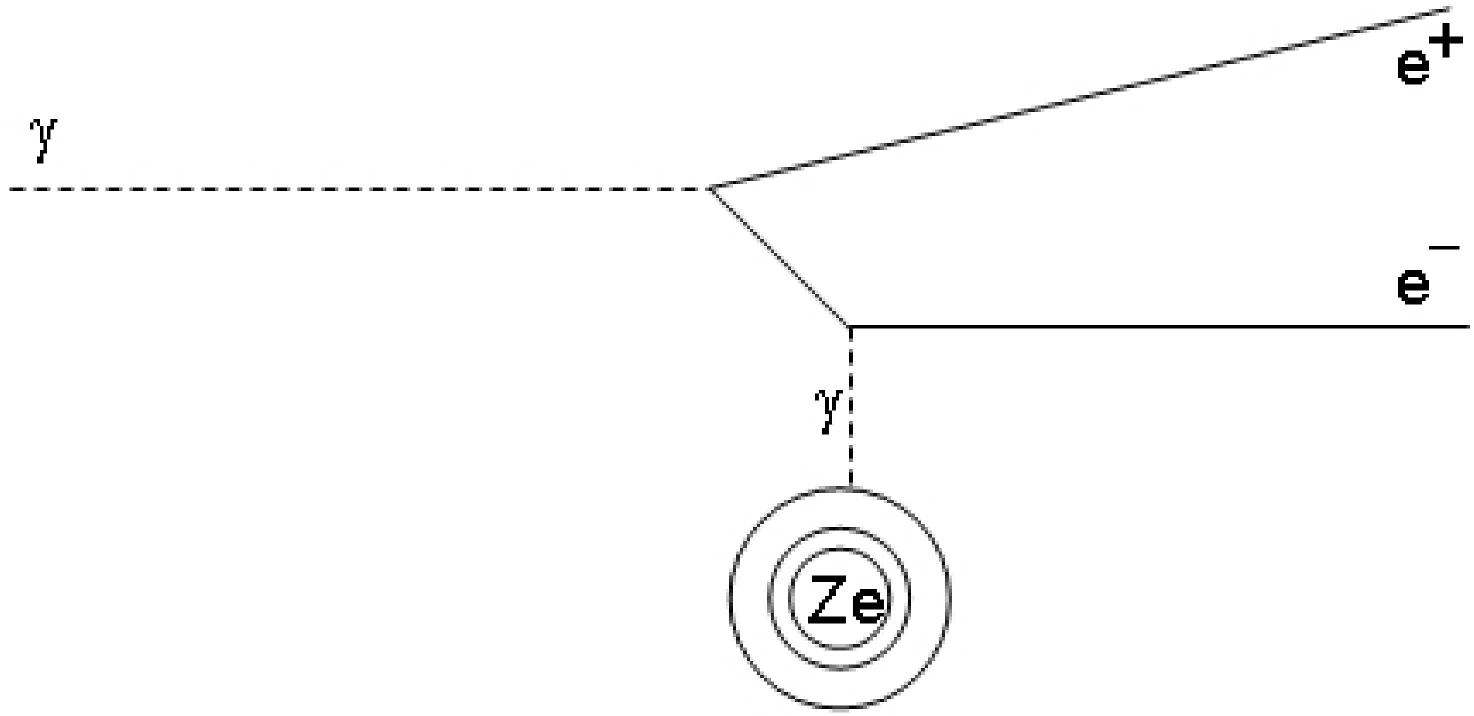


Electron-Positron Pair Creation

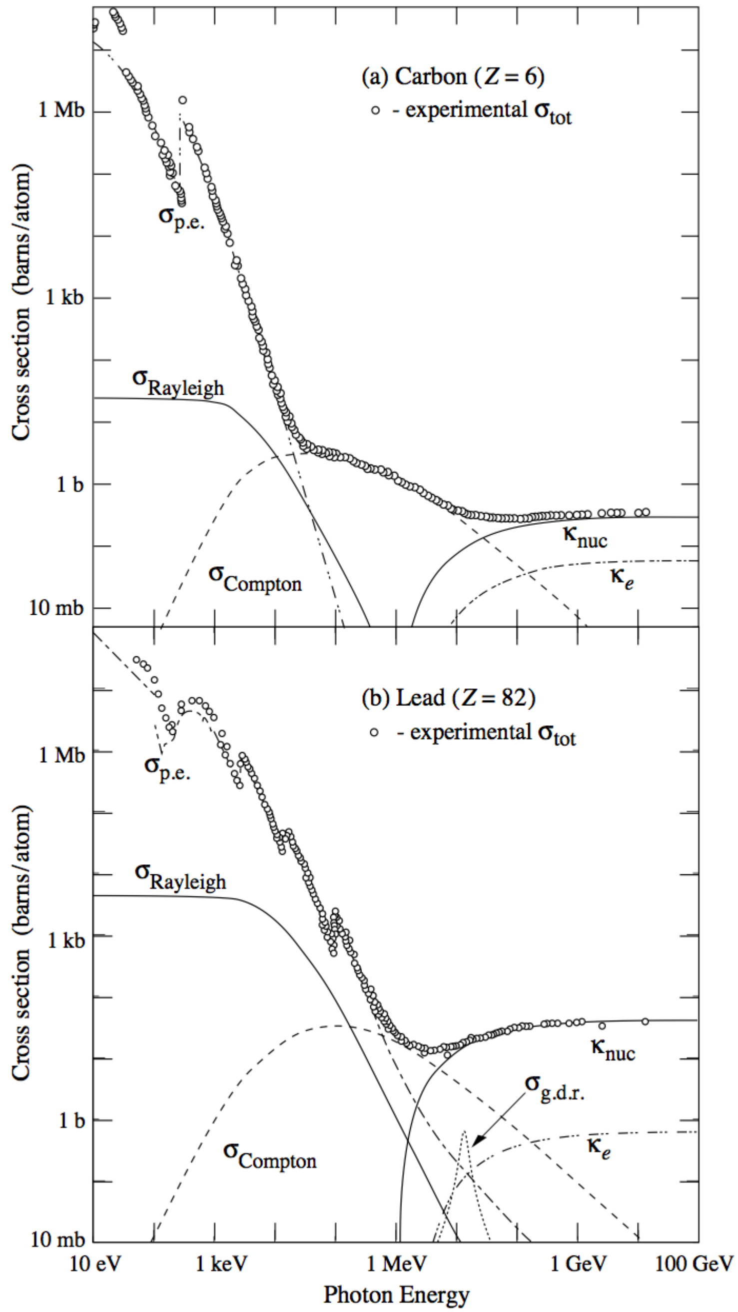




Momentum Conservation in Pair Prodcution



Photon Cross Sections (Carbon and Lead)



Advanced Lab Experiment