PHY293 Lecture #12

November 20, 2017

1. Light as a Wave **&** Particle
   - Classically think of light as a wave.
   - Maxwell's equations include a wave-equation that describes the propagation of EM fields
   - Over the last week we've seen several examples where light has a distinctly particle nature
     - Photo-electric effect
     - X-ray production
     - Compton scattering
     - Inelastic processes that absorb or emit a single photon
   - So which is it? As we'll see it's both !
     - The answer depends on what question you are trying to ask
     - Things get worse (next time) when we see particles (electrons, protons etc.) also exhibit this dual behaviour – there are instances where they behave more like a wave.
   - Wave or Particle? Anticipate which by comparing the wavelength to a typical dimension in the problem
     - If $\lambda \simeq D$: then it makes sense to think of light as waves
     - If $\lambda << D$: then better to think of light as a particle
   - Referred to as Wave-Particle Duality
2. Single Slit Diffraction
   - Light of wavelength λ passing through narrow slit, of width a, that is comparable to λ exhibits (Fraunhofer) diffraction
   - This is one manifestation of the wavelike behaviour of light
   - Note that smearing out of arrival point on screen is inversely proportional to a, the width of the slit
   - Condition for minima: rays 1 and 7 (see figure) must differ in path length by $\lambda/2$. Their path difference is $(a/2)\sin\theta$.
     - Conclude $a\sin\theta = \lambda$ is condition for destructive interference (MINIMUM!)
   - In this case rays 2/8 and 3/9 will also interfere destructively.
   - Consider an example: EM waves incident on a slit 1 μm in width and determine full angular width (from first minimum on one side to first minimum on the other – in degrees) of the central diffraction maximum
     - Do this first for visible light: 500 nm
       * Minima at $a\sin\theta = m\lambda$. First minima at $m = \pm 1$ so $\sin\theta = \pm\lambda/a = \pm 500/1000 = \pm 0.5$
       * This is not such a small angle: $\pm 30^o$.
       * Depending on how far away the screen is, the main diffraction peak will be **much** broader than the slit
       * Here it makes sense to talk about light waves producing an interference pattern
     - Do it also for an X-ray with $\lambda = 0.05$ nm
       * Same formula: $\sin\theta = \pm\lambda/a = \pm 0.05/1000 = \pm 5\times 10^{-5}$ radians $\Rightarrow \pm 0.003^o$
       * So the beam diverges hardly at all
       * Almost independent of how far away the screen is, the first diffraction peak will be $\approx$ the same width as the slit.
       * Here it makes sense to talk of light particles that travel through slit and continue (in parallel) to the screen
3. Double Slit Diffraction
   - Produces and even more conclusive demonstration of wave interference
   - Difference in path length (to screen) from the two slits is $d\sin\theta$ (d is the separation between the two slits, and the openings are assumed to be much smaller than d)
   - For constructive interference (MAXIMUM – this is a similar argument to the single slit pattern, but with the opposite outcome – maximum here) want this difference in path length to be equal to an integer number of wavelengths $m\lambda$
   - $y/D \approx \sin\theta$ and $d\sin\theta = m\lambda$

- So inference maxima appear at: $y = \frac{m\lambda D}{d}$ (assumes all are at small angles ($\sin\theta \approx \theta$)
- Full calculation gives: $I(\theta) = 4I_0\big(\frac{\sin(\pi b/\lambda\sin\theta)}{\sin\theta}\big)^2\cos(\frac{2\pi d\sin\theta}{\lambda})$
  - I_0 is the maximum light intensity coming through a single slit
  - The solid line is the single slit diffraction peak we talked about at first
  - For two slits, the light wave amplitude doubles and the intensity (amplitude ^{2}) goes up by a factor four
  - While there are minima in the two slit interference pattern at λ/d the overall 'peak' has the same width out to $\pm\lambda/b$ (b is distance between the two slits here, or the full width of the opening if it were just a single slit)
- Consider Problem 3.44 – not assigned (unlikely to see one like this on final exam...) but provides some additional context to these statements

4. Single Photon Interference?

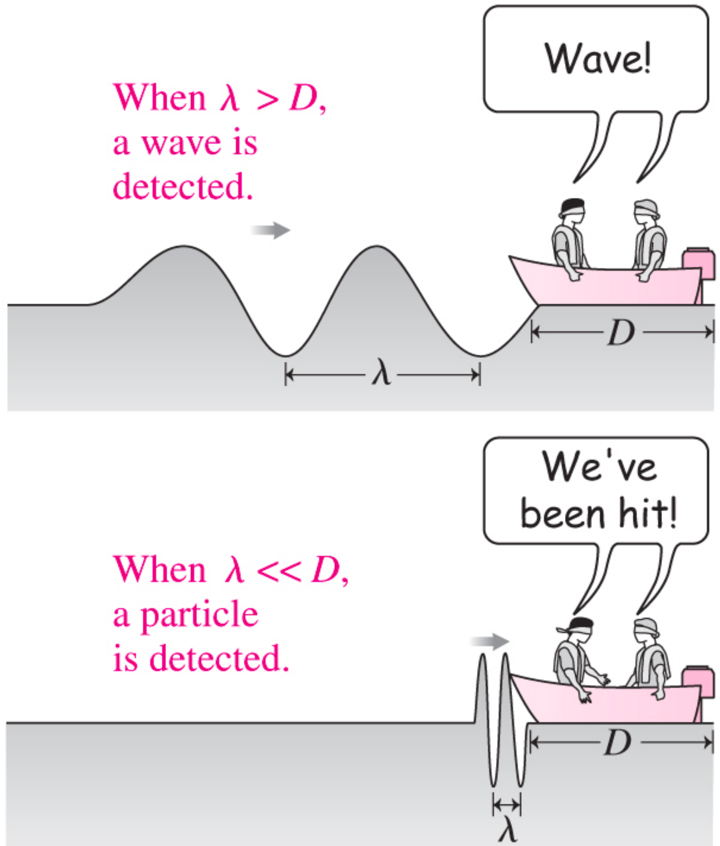
- What happens if we reduce the intensity of the light source so that only one photon 'at a time' passes through the slit(s)
- Consider single photons impinging on a double-slit apparatus (very dim source)
- First look at what the screen (beyond the slits) would look like after 10 photons detected – pretty random
  - no pattern to predict arrival of next photon
- Next look at 100 photons: still not much
- Maybe after 1000 photons start to see pattern. There are clearly favoured places and disfavoured places
- With 10000 photons pretty clear that there is an interference pattern even when the photons hit the slits **one photon at a time**
  - Is the photon passing through both slits?
  - Can a single photon interfere with itself?
- Despite unpredictable arrival of any single photon, we still end up with a regular pattern
- Something is controlling the probability that the next photon will arrive in particular location
- The observation is consistent with the Intensity (amplitude of EM wave – squared) being the predictor of the probability that photon will end up at a particular location on the screen.
- We refer to the EM waves as the "wave function" that describes the physical (electromagnetic phenomena) system and the square of the amplitude of this wavefunction (generically, we'll call it $\Psi(x)$) giving the probability that the next photon will arrive at position x : $P(x) = |\Psi(x)|^2$

5. Interpretation

- Quantum mechanics does not provide an answer to the question "Which slit did the photon pass through".
- Often hear: " You cannot ask QM which slit the photon passed through, other wise you change the physics".
- Indeed, if you put a detector next to each slit and try to figure out, on a photon-by-photon basis, which slit the photon is 'nearest' ... the interference pattern goes away.
- If you don't try to pin down the photons then QM says that, until it is detected at some location x on the screen, it has some probability to have passed through either slit and to have 'interfered with itself'.
- Once **detected** at screen we say the "wavefunction has collapsed" and there is 100 % probability that it was found at x

6. Summary

- Light appears to behave as both a particle and a wave
- Even when we look at it in the form of single photons it still exhibits wavelike behaviour
- The "quantum" of the electromagnetic field is the photon
- Interpret square of EM wave amplitude (at any point in space) as representing the probability to find a photon at that point
- What happens if we do the double-slit experiment with particles like electrons?
- Do other particles exhibit wavelike behaviour? See next lecture



$\lambda \ll D$: particle

$\lambda \gtrsim D$: wave

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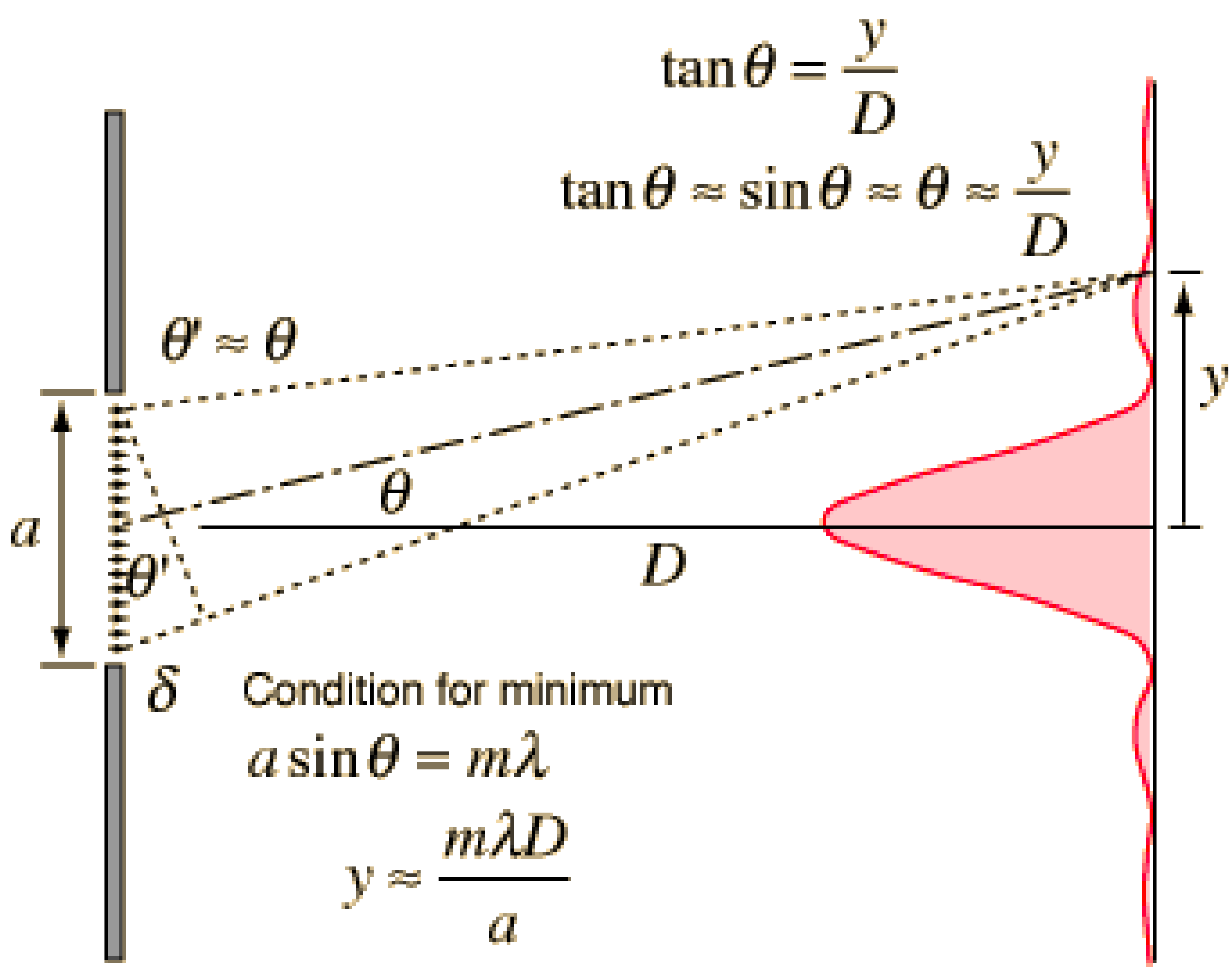
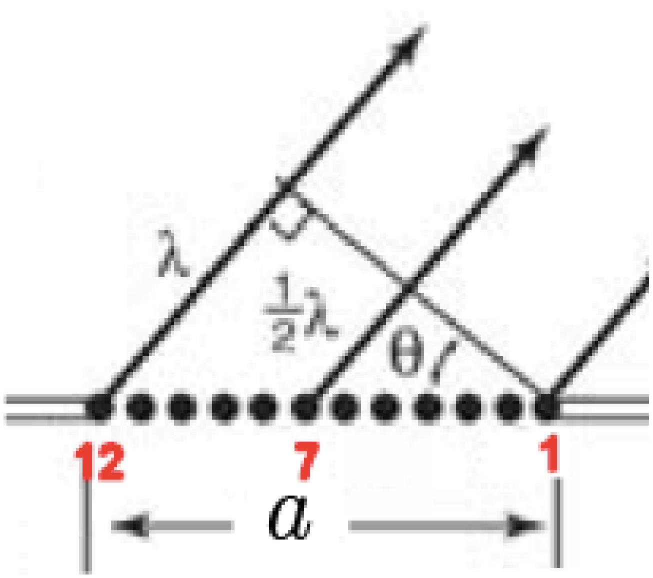
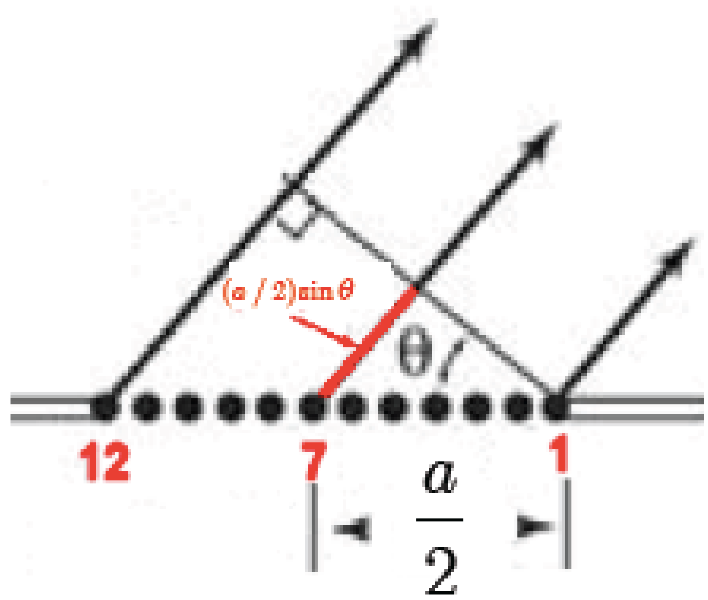
Single Slit Diffraction



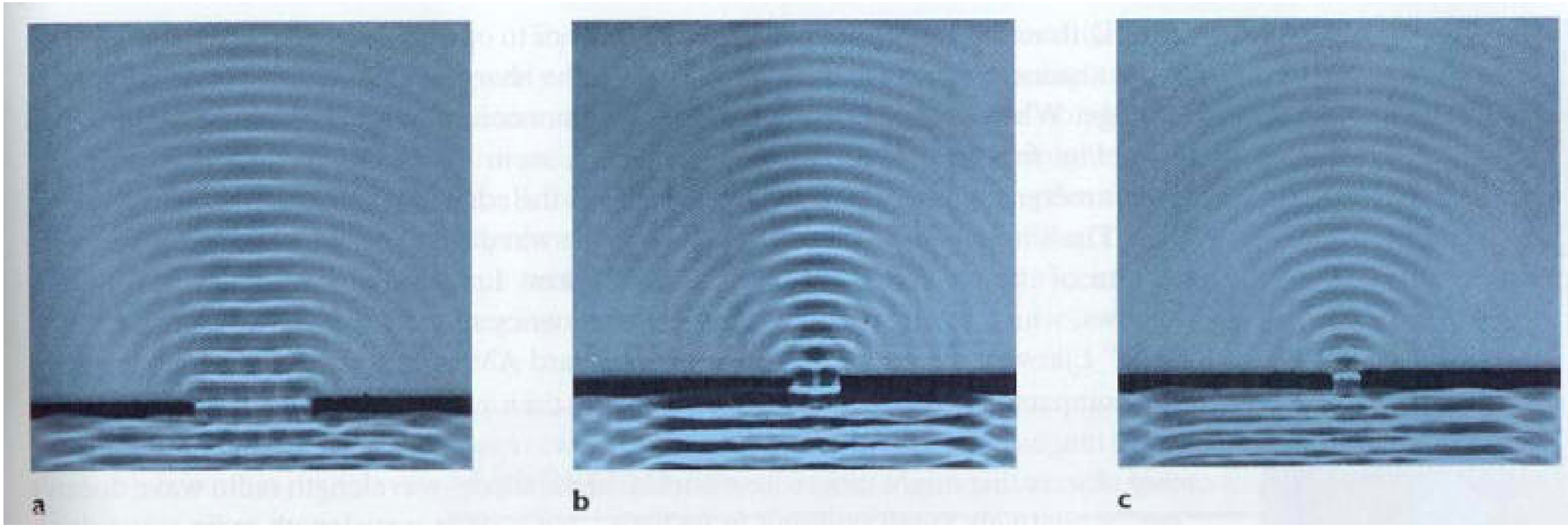
Interference Pairs



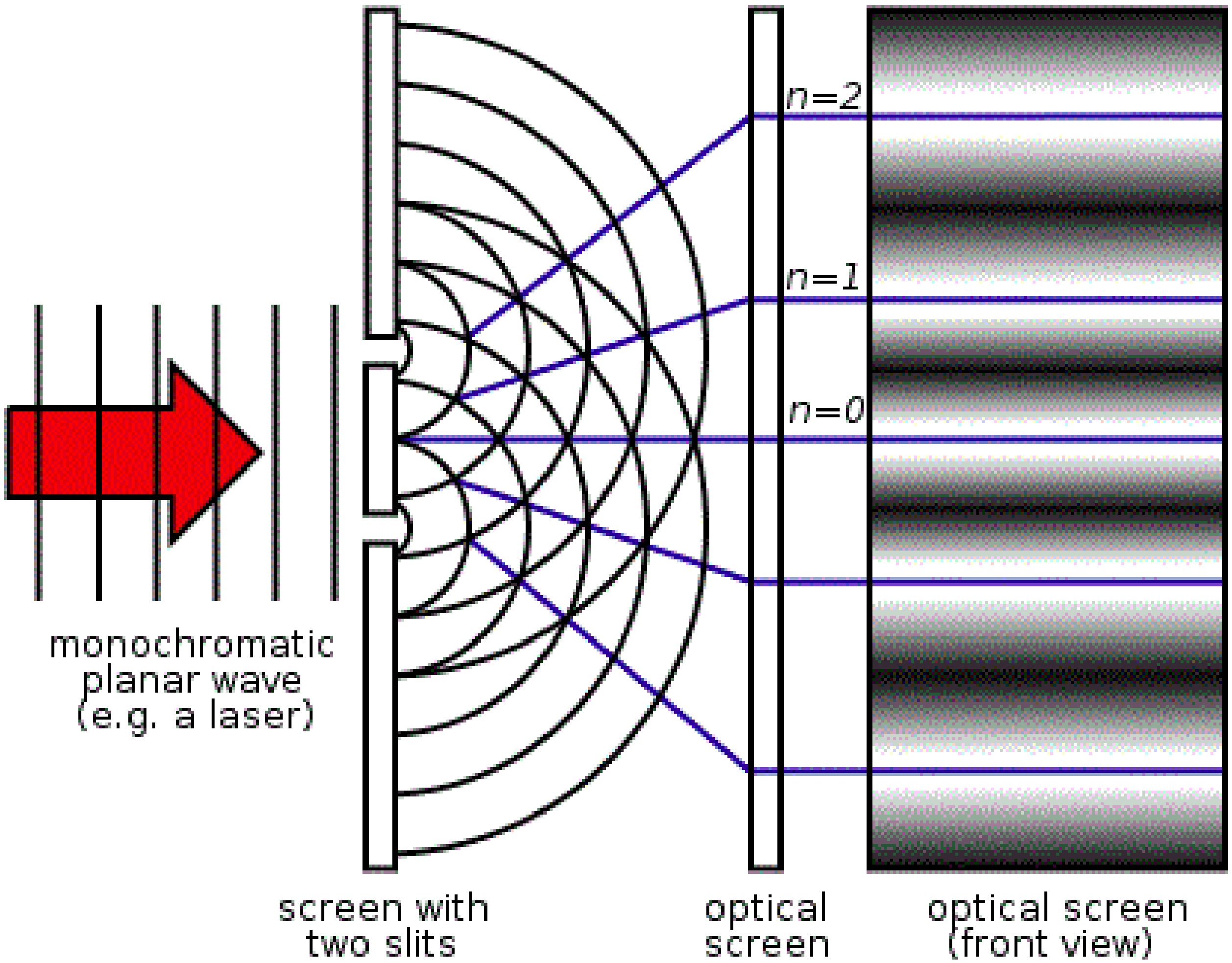


Water Waves emerging from Opening

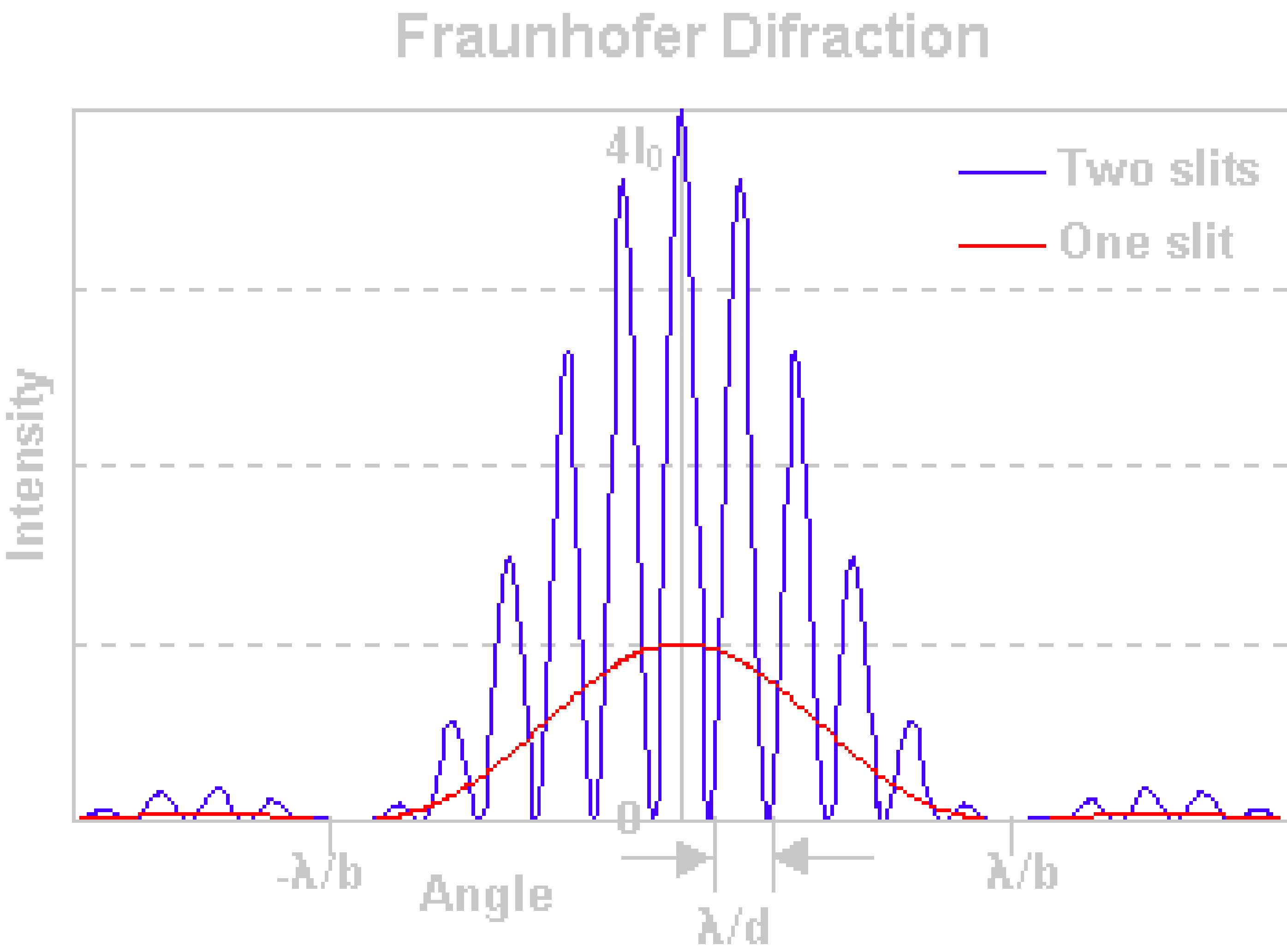
Controlled Water Waves



Double Slit Interference



Double Slit Calculation

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d = slit opening
b = distance between two slits
I0 = intensity with one slit open
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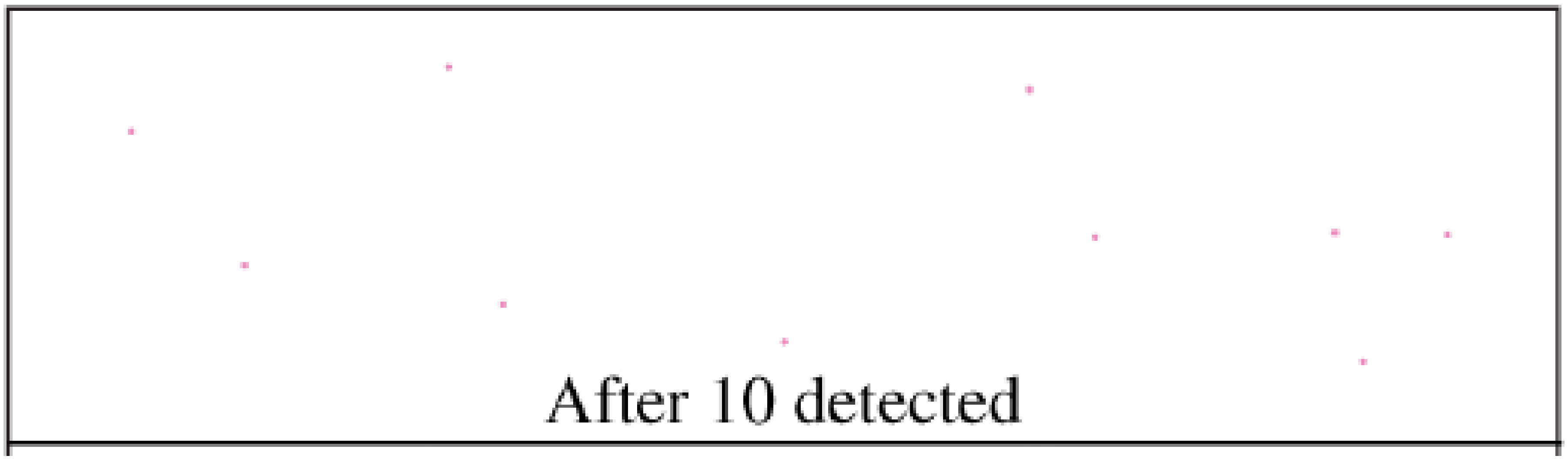
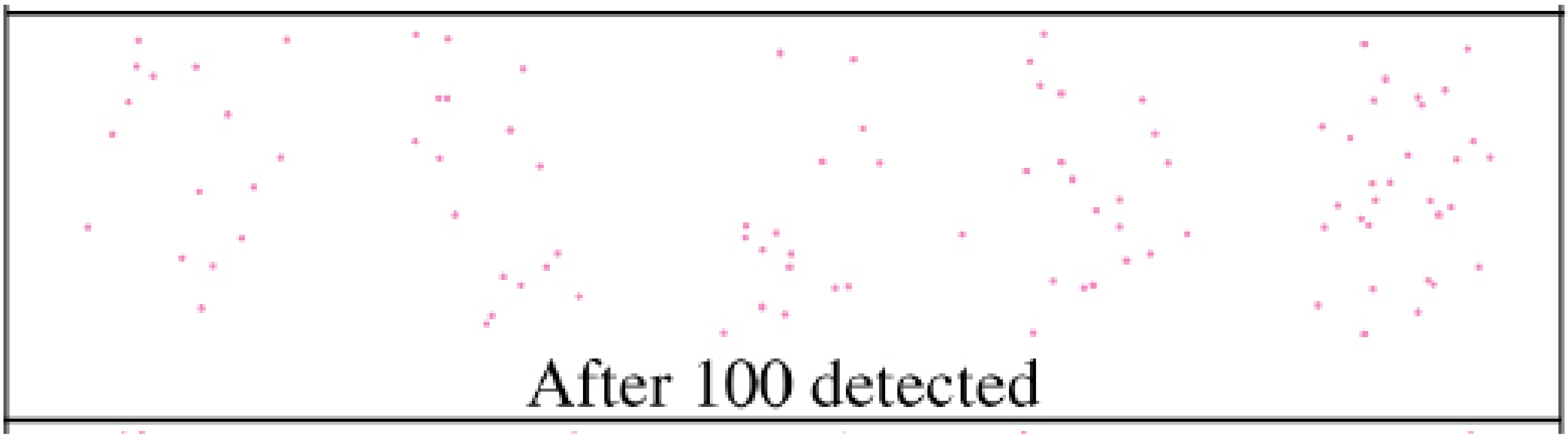
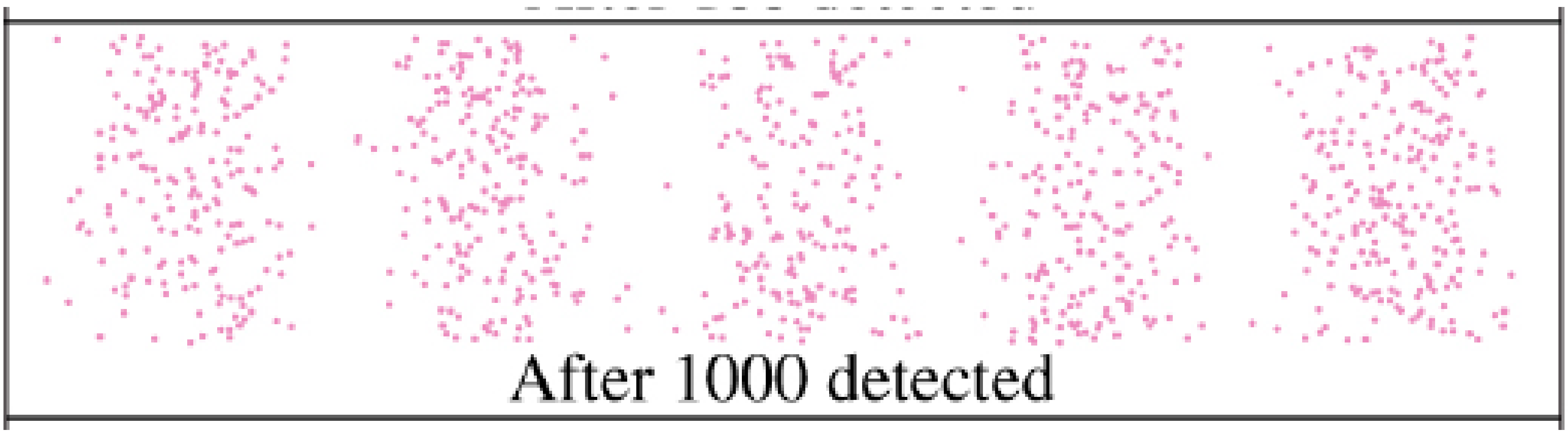
10 Photons



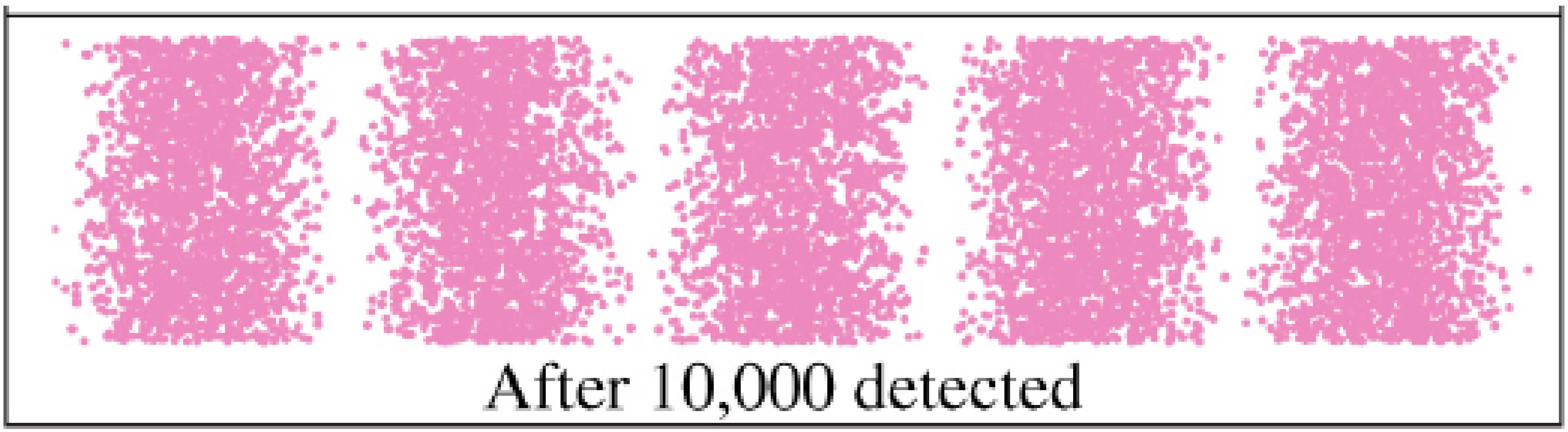
100 Photons



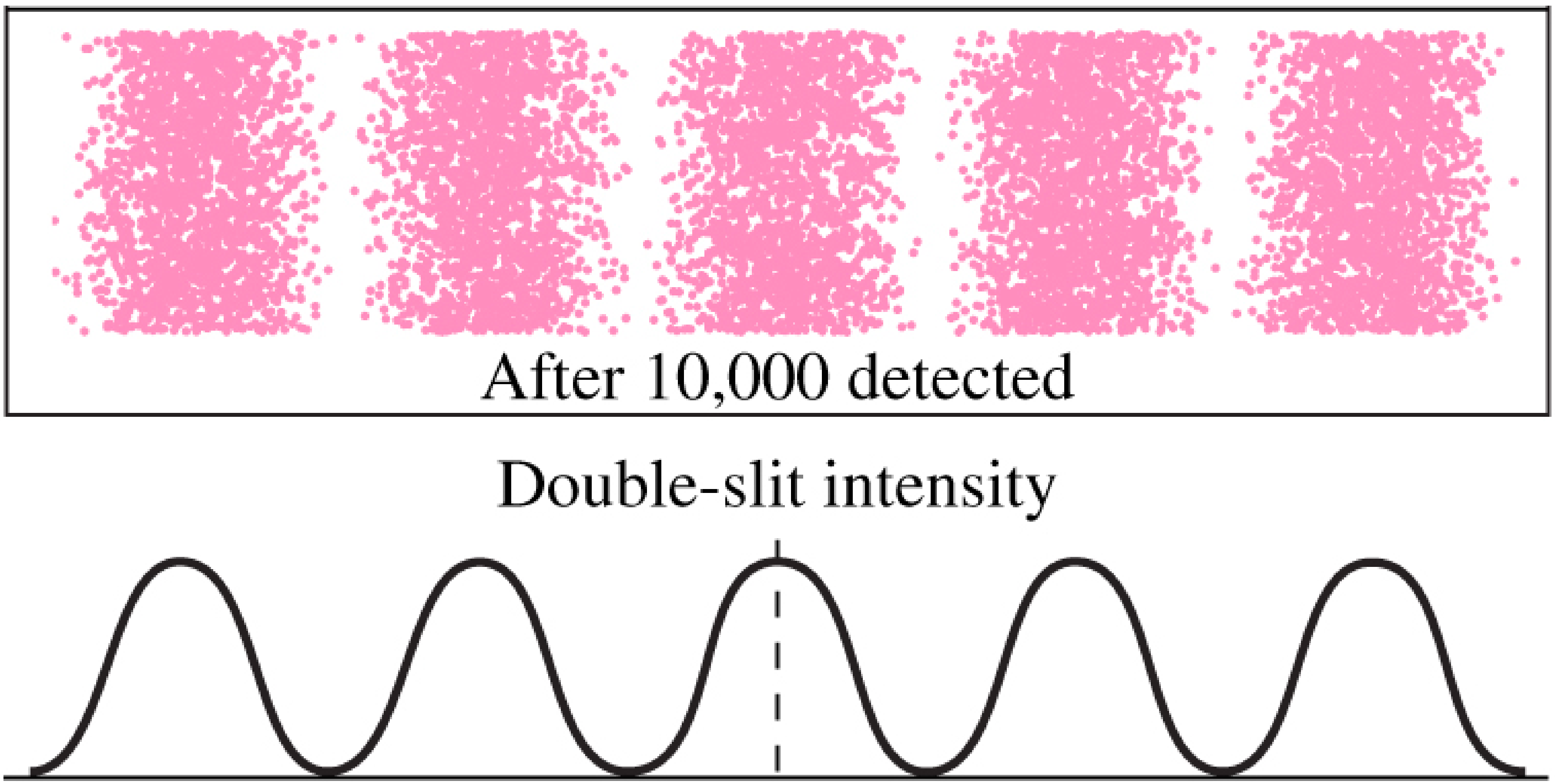
1000 Photons



10000 Photons



Photons with Intensity Map



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Wavefunction Collapse