PHY293 Lecture #13

November 21, 2017

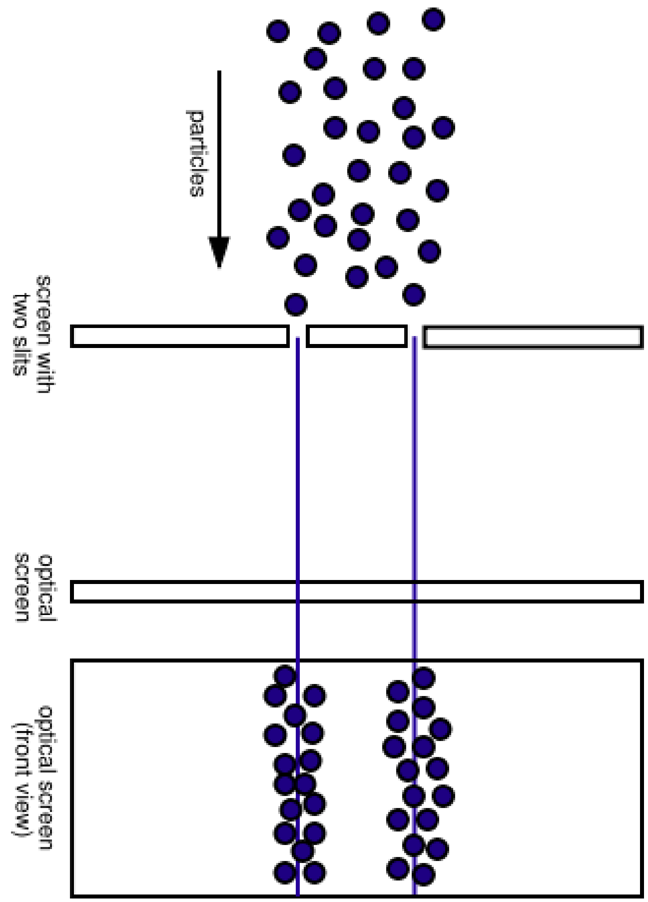
1. Particles Behaving as Waves?
   - Last time saw that photons, even when passing one-by-one, through a two-slit apparatus exhibit interference
   - Probability where photons end up described by a wavefunction – the EM wave amplitude from Maxwell's equations
     - At a given point, square of the wavefunction gives probability of finding the photon **if one were to make a measurement**
     - Making the measurement **collapses the wavefunction** making any subsequent predictions meaningless
     - Attempting to measure which slit the photon passes through, collapses wavefunction before interference can take place
   - Key concept: Photon **probability wave** passes through both slits and interferes with itself, results in classical wave interference pattern **even when incident light so dim than only single photons present in the apparatus at any given time**
2. Double slit experiment with Electrons
   - Imagine first a double slit experiment with classical particles (grains of sand?)
   - Particles pile up under the openings
   - Electrons that pass through a 'wide' slit produce bright spot roughly the same width as the slit
     - This also happens with light if slit is very wide (ie. light through a doorway doesn't diffract)
     - Still need to define 'wide' in the context of electrons.
     - For light it is the width of slit relative to the wavelength of light
   - Electrons passing through a single slit (either slit of a double slit apparatus (the other slit is covered)
     - Just spread out "all over" the screen
     - See what we mean by "all over" in a few minutes
     - But already this is reminiscent of the diffraction of light in a single (narrow) slit
   - Double slit experiment reproduces the results of light through a double-slit
     - There are minima (absence of electrons) at places where electrons **would arrive** if only one slit was open
     - Not expected classically – even if I've "set you up" to expect it by insisting on similarities between electrons and photons
     - And this interference pattern emerges even if electrons pass the double-slit barrier one at a time.
3. Hitachi: Biprism Double Slit Experiment with Electrons
   - The wire generates a magnetic field that bends electrons right or left (depending on their initial directions)
   - They curve around and, potentially, interfere with each other before arriving at detector plane
   - For more details see: http://www.hitachi.com/rd/portal/research/em/doubleslit.html
   - So electrons also exhibit wavelike interference – until 1920's no one thought to look on this scale
   - Single electrons, incident on a double-slit (like) apparatus produces an interference pattern
   - Conclusion similar to the one obtained from discussion of individual photons
     - There is a **wavefunction** associated with each electron
     - That wavefunction passes through both slits and interferes with itself
     - Producing a probability distribution for the arrival location of the electron on the detector plane
     - Interpret the square of the wavefunction $|\Psi(x)|^2$ as the probability to find an electron at position x
   - We know what an EM wave is – Can be observed with antennae etc. – provides electric and magnetic field amplitudes at any point in space
   - What are these matter waves? Can we see them? No one ever has
   - What is oscillating? Again, no ether.
     - Only answer: mathematical construct representing **probability** to an electron (or other particle) at any given point
     - In the case of the Hitachi experiment it is the probability that an electron will show up at a certain point across the screen
     - In an atom, it is the probability of finding an electron at a certain radius from the nucleus. This comes from the wavefunction $\Psi(\vec{r})$ (see more than one-dimensional wavefunctions in PHY294)

4. Applications of Electron waves
   - Used to study the crystal structure of materials
   - Electron microscope produces a collimated beam of mono-energetic electrons
   - A crystal lattice is similar to a two-slit grating
     - Electrons can penetrate many layers (atomic planes) of the crystal
     - Some reflect from each layer and the interference from different layers is a measure of the distance between layers
     - Also provides information on the distance between neighbouring atoms in a single layer
     - See for example the Bragg Condition $2d\sin\theta = n\lambda$ original derived for x-ray scattering off crystals but also applies to electron diffraction
   - But still haven't figured out what the relevant wavelength is ...
   - Already seen $p = h/\lambda$ follows from quantisation of light ($E = h\nu$) and energy/momentum relationship for light (ie. $E = pc$)
5. DeBroglie (1924) hypothesis: $p = h/\lambda$ for **any** object (not just light)
   - Presented his conjecture in his thesis
   - Davisson & Germer confirmed it with electron diffraction experiments in 1927
   - DeBroglie won the Nobel Prize for his work in 1929
   - Davisson and Thompson (independent observation) wont the 1937 Nobel prize
   - So what wavelengths does this give?
   - Compute this for a macroscopic object (baseball) and microscopic one (electron)
     - For a baseball, say travelling at 100 km/h $p = mv = (150\text{ g})(27.8\text{ m/s}) = 4.17$ kg m/s
     - DeBroglie would predict a wavelength of $\lambda = h/p = 6.63 \times 10^{-34}/4.17 = 1.59 \times 10^{-34}$ m
     - Even if we put this in nm (1.6×10^{-25} nm) it is pretty small
   - Typical atoms (that make up the baseball) are perhaps 0.1 nm across, so this wavelength is still 10^{24} times smaller
   - Interference between baseballs would be **very very** small. Conversely we'd expect macroscopic objects like this to pile up like classical particles if they pass through two slit apparatus – unless the slits were 10^{-25} nm across.
   - To get a deBroglie wavelength comparable to the atomic scale need a particle with very low momentum
   - Should have both very low mass and velocity
     - An electron at room temperature has kinetic energy given by $3/2k_bT$
     - At $T = 293$ K (room temperature) this gives $E = 1.5 \cdot 1.38 \times 10^{-23} \cdot 293 = 6.07 \times 10^{-21}$ J
     - Which in turn gives a momentum of $p = \sqrt{2mE} = \sqrt{2 \cdot 9.1 \times 10^{-31} \cdot 6.07 \times 10^{-21}} = 1.05 \times 10^{-25}$ kg m/s
     - Which finally gives $\lambda = h/p = 6.63 \times 10^{-34}/1.05 \times 10^{-25} = 6.31 \times 10^{-9}$m or about 6 nm
     - This is 60x larger than the typical atomic dimensions (1 Angstrom or 0.1 nm)
   - Room temperature neutrons/protons on the other hand have smaller wavelengths because they are much more massive
     - $p = \sqrt{2mE} = \sqrt{2 \cdot 1.7 \times 10^{-27} \cdot 6.07 \times 10^{-21}} = 4.5 \times 10^{-24}$ kg m/s
     - This would give $\lambda = h/p = 6.63 \times 10^{-34}/(4.5 \times 10^{-24}) \approx 0.15$ nm
     - Just about the size/spacing of atoms in a crystal lattice
     - Scattering of cold (room temperature) neutrons is a common tool for materials characterisation
6. The Davisson Germer experiment
   - Used 54 eV electrons scattered off Nickel
   - Condition for constructive interference is $2d\sin\theta = n\lambda$
   - What is the wavelength of the 54 eV electron?

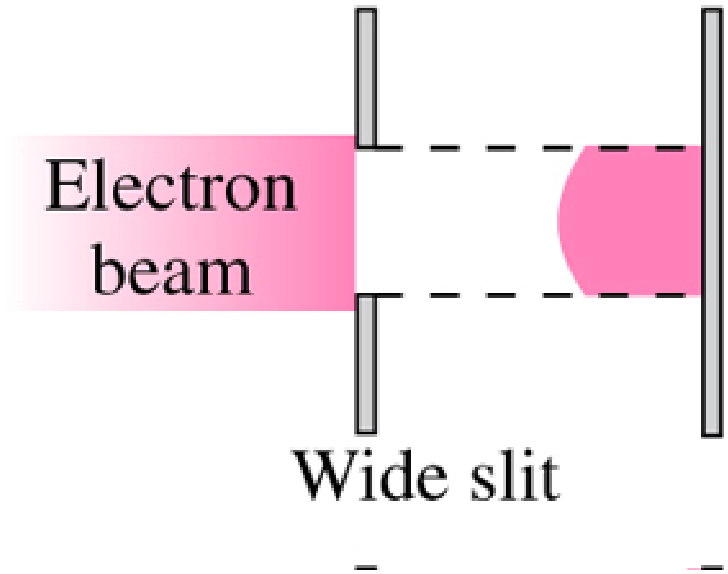
$$\lambda = h/p = h/\sqrt{2mE} = 6.63 \times 10^{-34}/\sqrt{2 \cdot 9.1 \times 10^{-31} 54 \cdot 1.6 \times 10^{-19}} = 1.2\,\text{nm}$$

   - Details of the 50 degree scattering angle $\Rightarrow \approx 0.5$ nm layer spacing in Nickel are less important.
   - Clear that O(nm) layers spacings in Nickel only lead to constructive interference if electrons behave as waves, with λ O(nm).
   - Davisson & Germer confirmed this experimentally.
7. What about when $v \to 0$
   - Then $p \to 0$ and $\lambda = h/p \to \infty$
   - Discuss the QM uncertainty principle next time – can't actually ever get $p \equiv 0$.

Interference of grains of sand

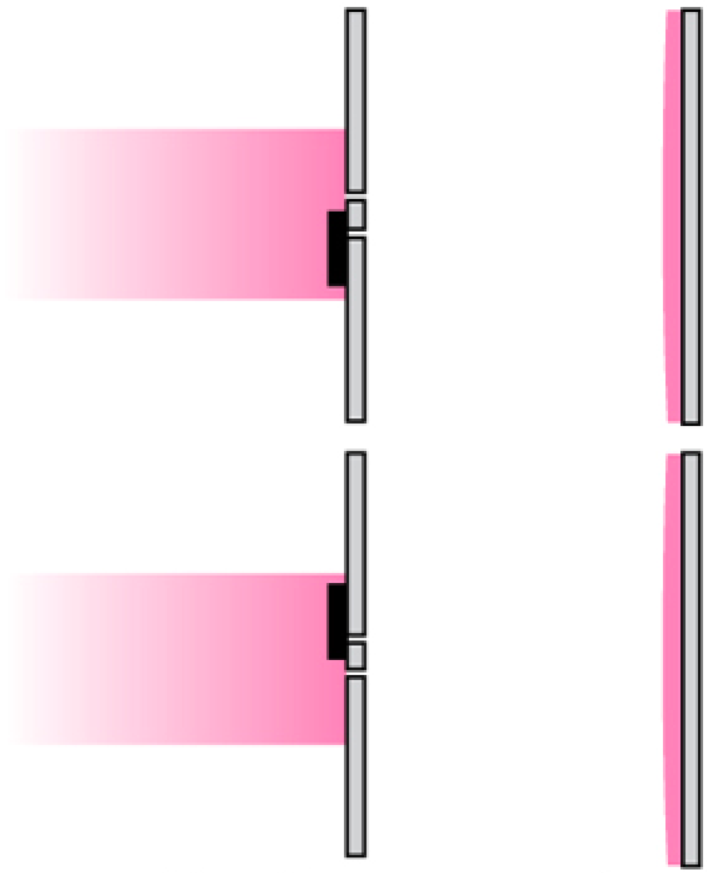


Electrons (Wide Slit)



Electrons passing through a wide slit arrive at a screen one by one and produce a bright region essentially the same width as the slit.

Electrons (Narrow Slit)

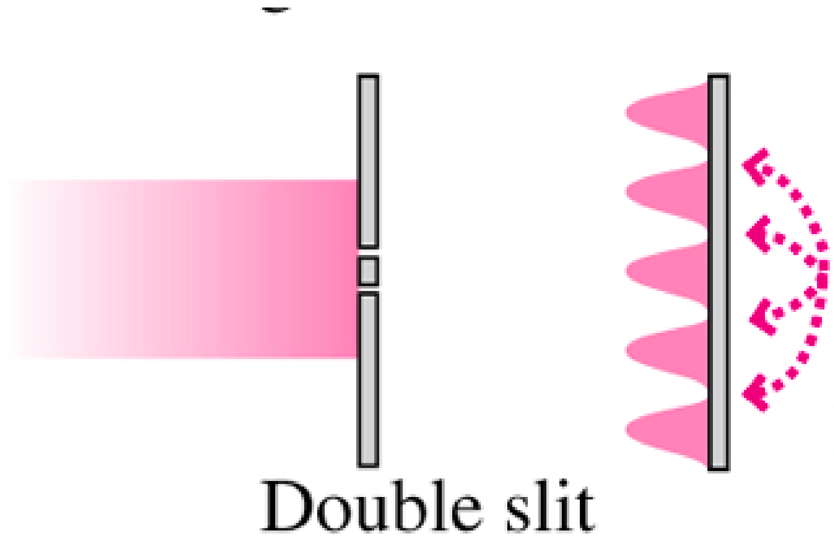


Single narrow slit

With a single narrow slit, electrons arrive one by one all over the screen. They don't avoid any points . . .

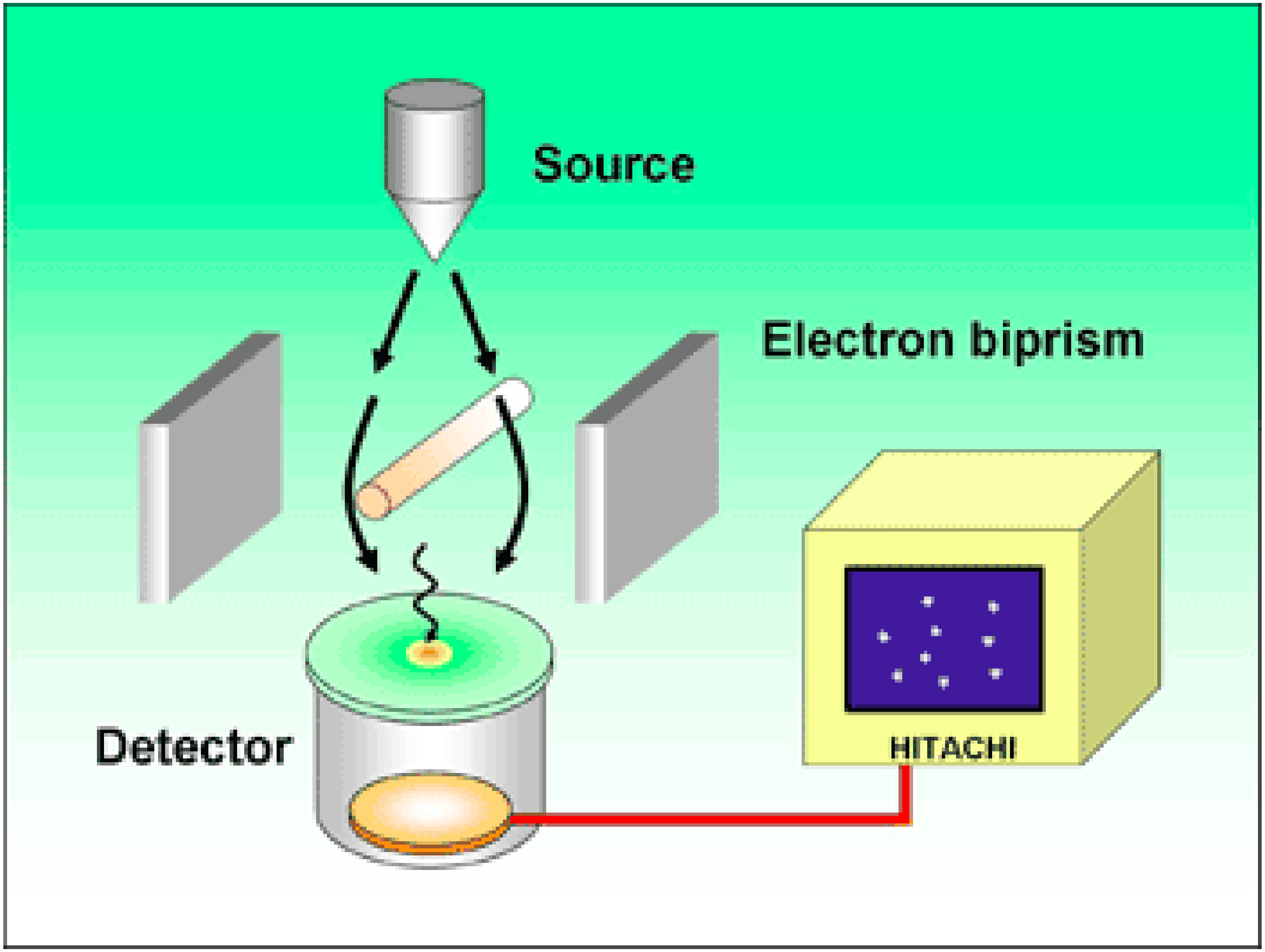
. . . but when *two*

Electrons (Double Slit)



. . . but when *two* slits are open, there are certain points at which no electron ever arrives. Each electron wave is destructively interfering with *itself*.

Electron Biprism Aparatus



Electron Arrival Pattern

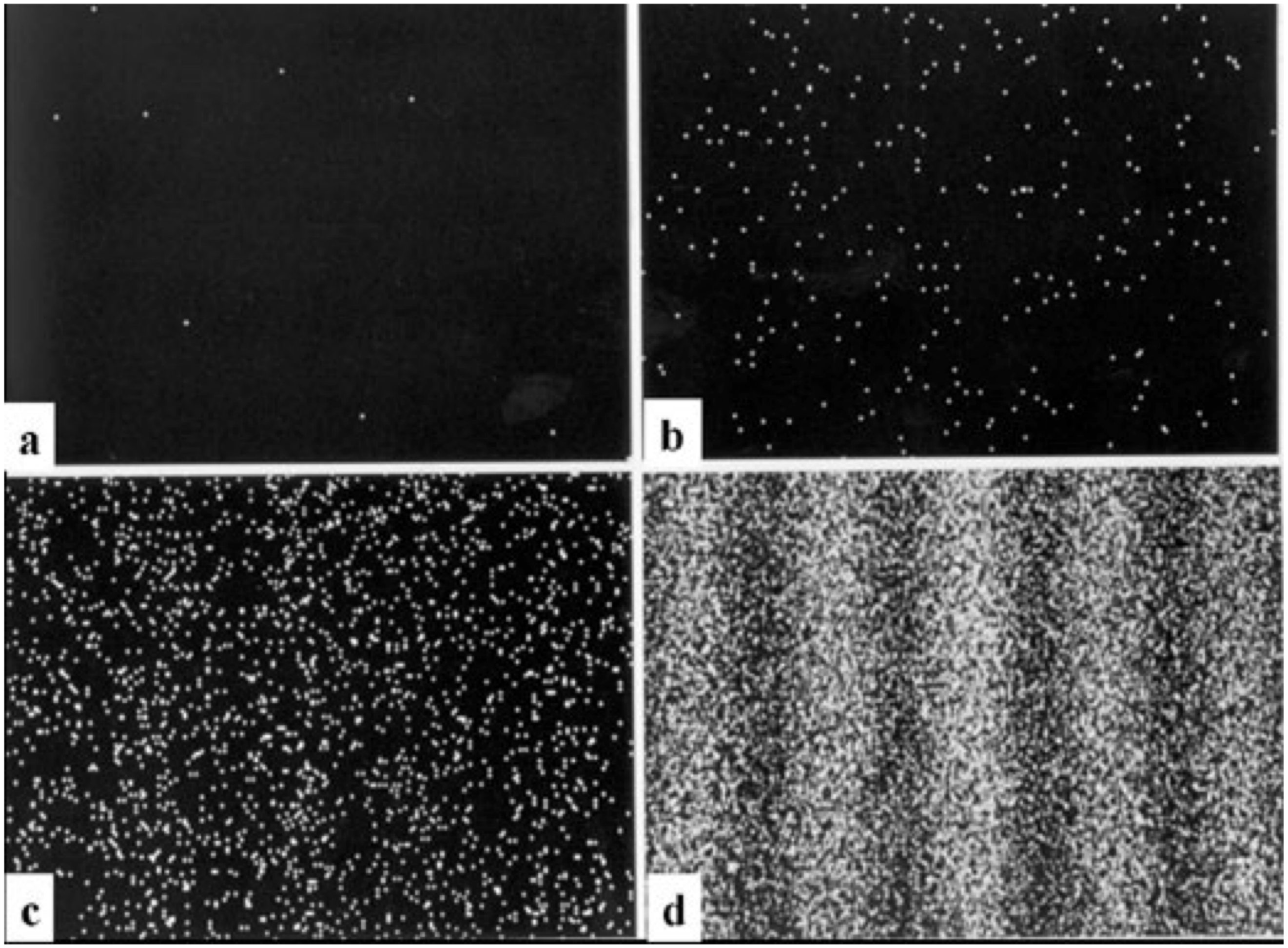
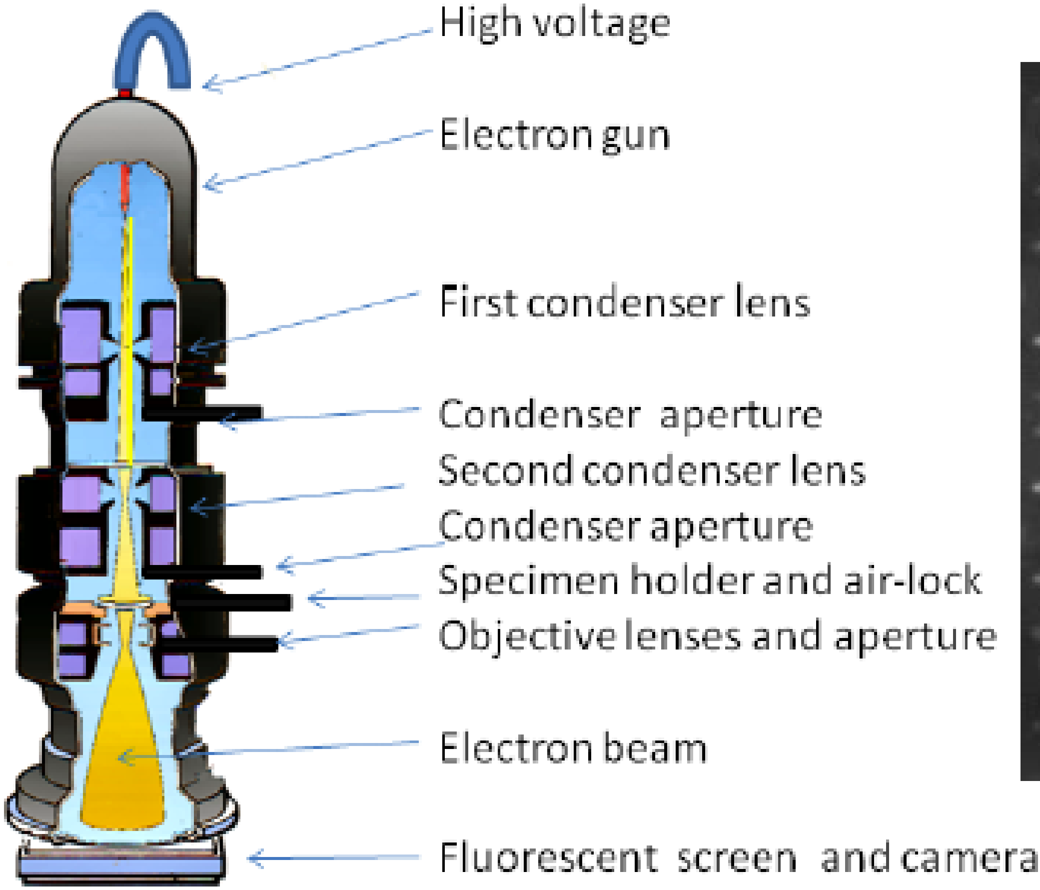


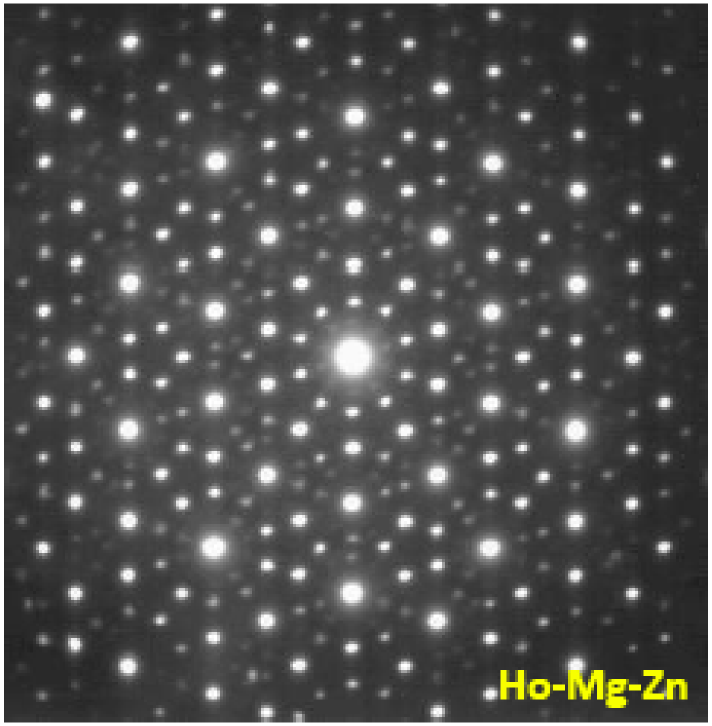
Fig. 2 Single electron events build up to from an interference pattern in the double-slit experiments.

The number of electron accumulated on the screen. (a) 8 electrons; (b) 270 electrons; (c) 2000 electrons; (d) 160,000. The total exponsure time from the beginning to the stage (d) is 20 min.

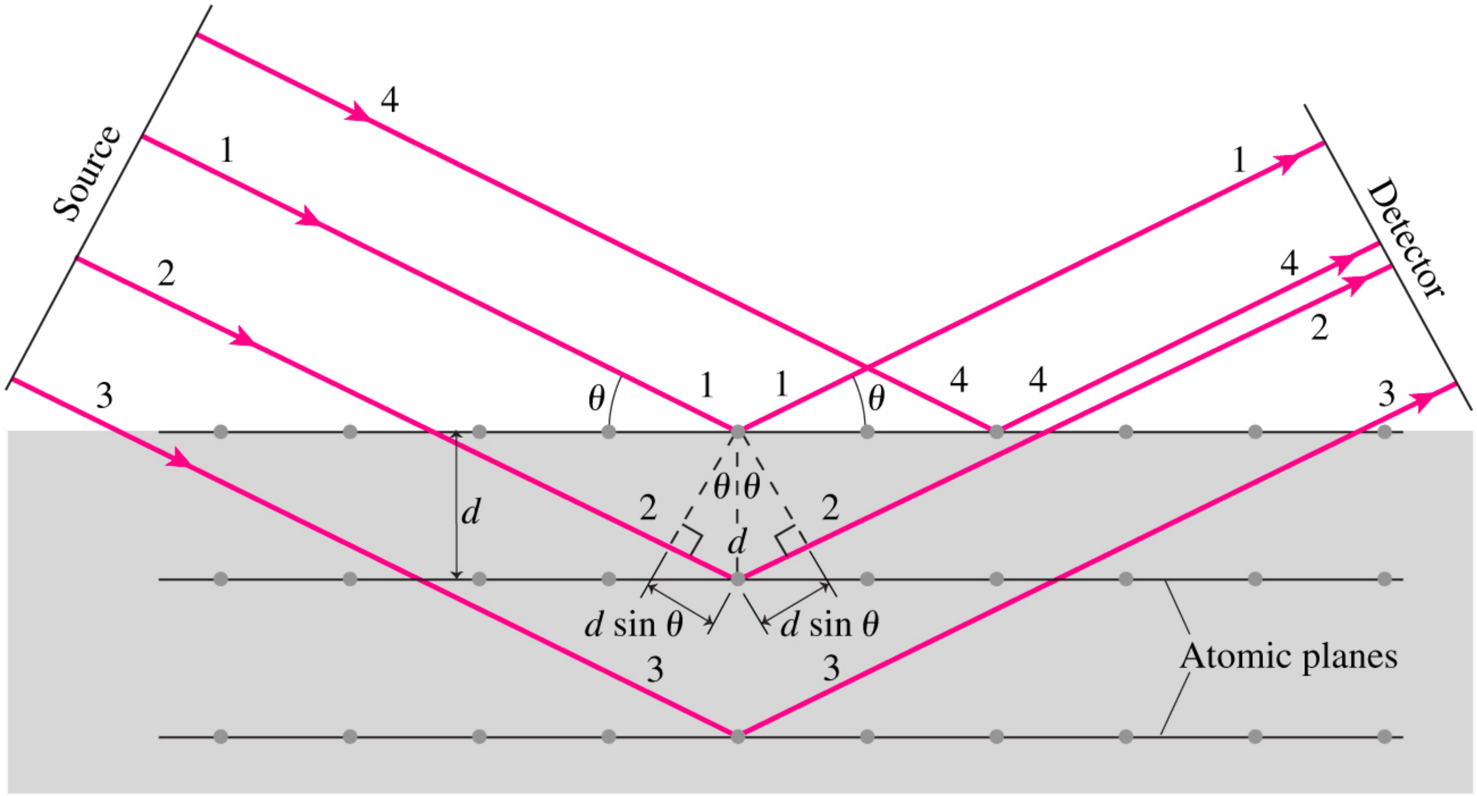
Electron Microscope



Transmission Electron Microscope



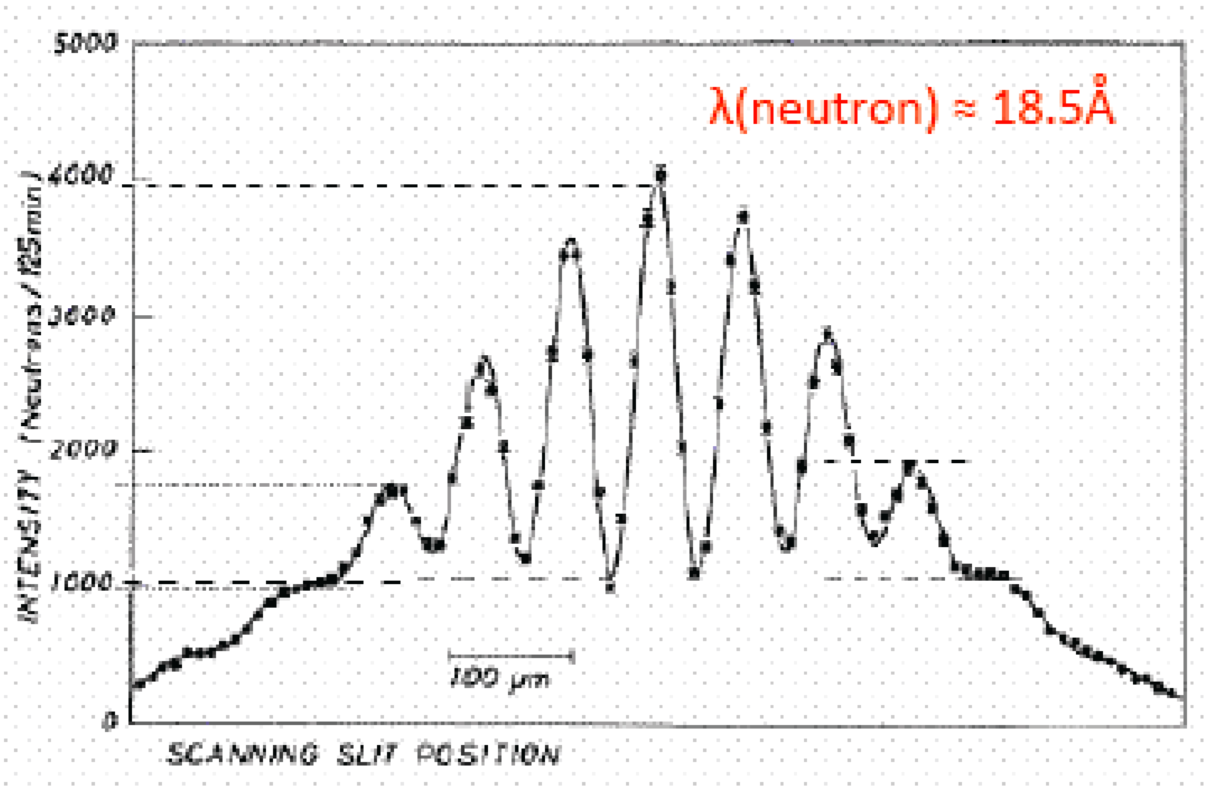
Electron Beam Diffraction



Neutron Scattering

Single- and double-slit diffraction of neutrons

Zeilinger et al., Reviews of Modern Physics, Vol. 60, No. 4, October 1988



Davisson and Germer