# PHY293 - Problem Set #4

November 7, 2017

1. **Problem 1 - 2.23** Let Anna's frame be $S'$ and Bob's be $S$. Here are the facts of the problem:

   - As Anna's COM (center-of-mass) passes Bob, both their clocks read 0.
   - The flasubulb goes off at $t' = 100$ ns (on Anna's watch.)
   - The time of this event according to Bob differs by 27 ns.

   (a) Does Bob's watch read 73 ns or 127 ns?

   - Let's use the Lorentz transformations. (Note, I have used the format assuming Anna is moving to the right of Bob, so $v > 0$.)

   $$t = \gamma\left(\frac{v}{c^2}x' + t'\right)$$

     where $\gamma = 1/\sqrt{1 - v^2/c^2} = 1/\sqrt{1 - 0.6^2} = 5/4$, $v = 0.6c$ and as stated in the problem $t' = 100$ ns. The last quantity to interpret is $x'$. In Anna's frame, her COM is at the origin, but her wrist where the flashbulb is located *is not.* So, we can interpret $x'$ as the length of Anna's arm (which is the subject of part (b)).
   - Since $x' > 0$ (the question says her arm is extended in the direction of motion), and $v/c^2 > 0$ and $t' > 0$, it follows that Bob's $t$ must be *greater* than $t'$. Thus, it can be concluded that *more* time elapses on Bob's watch, so $\boxed{\text{it's 27 ns LATER}}$.

   (b) How long is Anna's arm?

   - Again, use the Lorentz transformation shown above to get $x'$, which represents the proper length of Anna's arm. Isolating for $x'$ in the above Lorentz transformation, we get:

   $$x' = \frac{c^2}{v}\left(\frac{t}{\gamma} - t'\right)$$

     and putting in values (remembering that 127 ns $= 127 \times 10^{-9}$ s) we obtain $\boxed{x' = 0.8 \text{ m}}$, which is the length of her arm.

2. **Problem 2 - 2.25** Let Anna's frame be $S'$ and Bob's be $S$. Here are the facts of the problem:

   - The diagram shws how Bob views Anna's ship.
   - As the backs of the ships pass one another, both clocks (at the backs of the ships) read 0.
   - Bob Jr. is in Bob's ship, aligned with the very front of Anna's ship, as he perceives it.

   (a) In relation to his own ship, where is Bob Jr?

      - In Bob's frame, Anna's ships is length contracted, so Bob Jr is a distance of the length contracted ship away from the back of his own ship. This can be seen from the Lorentz transformation.

        $$x' = \gamma(x + vt)$$

        We know that the measurements [of distance] for the front and back of Anna's ship occur simultaneously so $t = 0$. We are left with

        $$x = \frac{x'}{\gamma}$$

        where $\gamma = 5/3$ and $x' = 100$ m (i.e. the proper length of Anna's ship), so we end up with $\boxed{x = 60 \text{ m}}$. Bob Jr. is 60 m from the back of his own ship.

   (b) What does the clock (at the front of Anna's ship) that Bob Jr. sees read?

      - Use the Lorentz transformations. We seek $t'$ given $x = 60$ m and $x' = 100$ m and $t = 0$. [Slight aside: to perhaps more intuitively think about this problem, one can note that while all the clocks in Bob's frame are synchronized to zero, i.e. at the front, middle, back of the ship, Bob and Bob Jr. *will NOT* perceive the clocks in Anna's ship to be synchronized. So when Bob and Bob Jr. think that it's $t = 0$, they will see only the clock at the back of Anna's ship reading 0. The other clocks, they will see will be slightbly behind.)

        $$t' = \gamma\left(-\frac{v}{c^2}x + t\right)$$

        Subbing in the values above, we find $\boxed{t' = -267 \text{ ns}}$.

3. **Problem 3 - 2.27** Let Anna's fram ebe $S'$ and Bob's be $S$. Here are the facts of the problem:

   - Let the two ends of the garage be such that Bob is on the LHS and Bob Jr. is at the RHS.

- The garage is of length 25 m, in Bob/Bob Jr. frame. That is to say, its proper length is 25 m.
- Anna's ship has a proper length of 40 m and is travelling from right to left (negative velocity).
- When Anna, at the front of her ship, passes Bob, both their clocks read 0.
- When Bob Jr's clock reads 0, the back end of Anna's ship passes him.

(a) How fast is Anna's spaceship moving?

- Let's use our old friend the Lorentz transformation.

$$x' = \gamma(x + vt)$$

(Please note that I have put a plus sign next to v since the ship is travelling in the negative x-direction). We know the measurements of length both occur at $t = 0$ for Bob and Bob Jr. We are left with

$$x' = \gamma x$$

We now need to solve for $\gamma$. This leaves us with

$$\gamma = \frac{x'}{x} = \frac{40}{25} = \frac{8}{5}$$

Knowing that $\gamma = 1/\sqrt{1 - v^2/c^2}$ some elementary algebra yields $\boxed{v = 0.781c}$.

(b) What will Anna's clock read when she sees the tail of her spaceship at the doorway where Bob Jr. is standing?

- Before doing the calculation, I think it's good to take one second and think about this intuitively. According to Bob/Bob Jr., Anna's ship is length contracted, so that's why it fits in the garage. But according to Annay, she sees that nose of her ship pass through the far door (i.e. where Bob stands) and then some time later, she sees her tail pass through where Bob Jr. is standing. (She would also perceive a length contracted garage, but in this case it is irrelevant here.) So, we are looking for the time it takes for the back of the ship to reach Bob Jr.'s door, according to Anna.
- We're a one trick pony. Lorentz transformation. We want $t'$ when $x' = 40$ m (i.e. the tail of Anna's ship), $x = 25m$ (i.e. when the tail of Anna's ship is at Bob Jr's door) and $t = 0$ (we know this occurs at $t = 0$ in Bob/Bob Jr's frame).

$$t' = \gamma\left(\frac{v}{c^2}x + t\right)$$

(Again, note the sign convention on $v$). Plug in $\gamma = 8/5$, $v/c = 0.781$ to get $\boxed{t' = 104 \text{ ns}}$.

(c) How far will Anna say the front of her ship is in front of Bob at this time?

- Let's take a second to ask ourselves what this question is really asking. When Anna perceives the back of her ship to have crossed Bob Jr, the front of her ship will have passed Bob. The question is, at what position does Anna perceive Bob?
- Lorentz. Transforms. Solving for $x'$ given that $x = 0$ and $t' =$ 104 ns.

$$x = \gamma\,(x' - vt')$$

Since $x = 0$, the $\gamma$ cancels out and we're left with

$$x' = vt'$$

and taking $v = 0.781c$, we obtain $\boxed{x' = 24.4 \text{ m}}$.

- A small aside: this solution may appear confusing because I put in all values of $v$ as $+0.781$ but incorporated the direction by changing the sign in the Lorentz transformations. I'm pretty sure I did everything right and it works out, but maybe I made a little sign error. Just be aware of this here...

4. **Problem 4 - 2.30** Let your vehicle's frame be $S'$ and the gas station frame to be $S$. Here are the facts of the problem:

- In your car, you're moving forward at 20 m/s
- There is a gas station at both $x = 0$ and $x = 900$ m.

(a) Is the clock at the second gas station at $x = 900$ m ahead or behind according to you as you pass it?

- This is the hard part of the problem, as the second part is just Lorentz transformations. To figure this out, put a flashbulb in between the two gas stations at $x = 450$ m. Set it off. In the gas station frames, the light will hit both gas pumps simultaneously. But remember how in SR if in one frame two events are simulataneous, they may not be in an other? So, consider now what happens in the car. Since the car is moving towards the gas station at $x = 900$ m, you in the car will perceive the light to hit the gas station at $x = 900$ m first. So this suggests that the clock at the second gas station is $\boxed{\text{ahead}}$.

(b) By how much?

- I think the easiest way to solve the question is as follows. We know that the clock at the second station is going to be ahead of yours. We know clearly that in the gas station's frame, that the journey will take $t = 900 \text{ m}/20 \text{ m/s} = 45$ s. So, find out what time your clock would read when this occurred.

- Lorentz transformations. $t = 45$ s, $x = 900$ m, $x' = 0$ (you're in your vehicle). Get $t'$.

$$t' = \gamma \left( -\frac{v}{c^2} x + t \right)$$

Calculating $\gamma$ (use a computer), we get $\gamma$ =1.000 000 000 000 002 200. Solving for $t'$, taking $v = 20$ m/s, we obtain $t'$=44.999 999 999 999 901 s. implying that the clock is ahead by $\boxed{1 \times 10^{-13}}$ s.

5. **Problem 5 - 2.40** Let the lab frame be $S$ and the muon frame be $S'$. Here are the facts of the problem:

   - The muon travels at $v = 0.94c$ and it lives for $0.032 \times 10^{-6}$ s in the lab frame.

   (a) How far does the muon travel in the lab frame?

   - Easy. We know the time and velocity in the lab frame. $d = vt = 0.94 \times c \times 0.032 \times 10^{-6}$ s $= \boxed{9.024 \text{ m}}$.

   (b) In its own frame, how long did the particle survive?

   - We want to know the lifetime of the particle in its own frame $t'$ given $t$. Use the Lorentz transformation.

   $$t = \gamma \left( \frac{v}{c^2} x' + t' \right)$$

   So, the particle is always at $x' = 0$ in its own frame, so this leaves us with $t = \gamma t' \implies t' = t/\gamma$. Simple algebra gives $\gamma = 2.931$, so we end up with $\boxed{t' = 11 \text{ ns}}$.

   (c) According to the particle, how long is the lab frame?

   - Similar to part (a), we now know a time in the muon's frame, and we know its velocity, so we can simply say $x' = 0.94c \times 11$ ns $=$ $\boxed{3.08 \text{ m}}$. This is the same as the length contracted distance corresponding to the length of the apparatus.

6. **Problem 6 - 2.46** Let Anna's frame be $S'$ and Bob's be $S$. Here's what we know about the problem:

   - Anna is born just as her spaceship passes Earth flying at 0.9c away.
   - Bob is born just as Anna passes Earth. This is equivalent to saying that both their clocks are synchronized to 0 as Anna passes Bob on Earth.
   - Planet Z is $30cy$ away according to Bob.

   (a) As Anna passes Planet Z, what is Bob's age according to Bob?

   - This one is easy as we have a measurement of distance and speed in the same frame. $t = 30cy/0.9c = \boxed{33.33 \text{ y}}$.

(b) What is Bob's age according to Anna?

- First, we need to know how much time has elapsed on Anna's watch. Use the formula for time dilation to solve for $t'$ given that $t = 33.33y$.

$$t' = \frac{t}{\gamma}$$

$\gamma = 2.2942$, and we find $t' = 14.53y$. This is Anna's age according to Bob. (Solution to part (d)). Now to know Bob's age according to Anna, we need to use the time dilation formula again, since Anna sees Bob receding at -0.9c. So, Bob's age according to Anna is $14.53y/\gamma = \boxed{6.33y}$

(c) What is Anna's age according to Anna?

- Anna's length contracted distance that she has traveled is $d' = d/\gamma$, so the time it takes her to travel this distance is $t' = d'/v = d/(\gamma v) = \boxed{14.53y}$.

(d) What is Anna's age according to Bob?

- Already done in part (b), $\boxed{14.53y}$.

7. **Problem 7 - 2.53** The source wavelength is 532 nm.

(a) As it moves along a line connecting Earth and the source, the observers on Earth see a wavelength of 412 nm. At rest, the source emits at 532 nm. What is the source's velociy?

- The relevant formula here is

$$f_{obs} = f_{src} \frac{\sqrt{1-\beta^2}}{1+\beta\cos\theta}$$

where I have simply used $\beta = v/c$. In this problem, since the source is blueshifted, the source is approaching Earth, so $\theta = \pi$.

- Using $c = \lambda f$ for light, this gives us

$$\frac{1}{\lambda_o} = \frac{1}{\lambda_s}\sqrt{\frac{1-\beta}{1+\beta}}$$

and after a little bit of mathematical jiggery pokery, we obtain

$$\beta = \frac{\lambda_0^2 - \lambda_s^2}{\lambda_0^2 + \lambda_s^2}$$

- Solving for $\beta$ using the above information, we obtain that $\beta = -0.25$, so the velocity is $\boxed{\text{0.25c towards the Earth}}$.

(b) What if it moved in the opposite direction? What would be the observed wavelength?

- Using $c = \lambda f$, the Doppler shift equation becomes

$$\frac{1}{\lambda_o} = \frac{1}{\lambda_s}\frac{\sqrt{1-\beta^2}}{1+\beta\cos\theta}.$$

- Taking $\theta = 0$, and solving for $\lambda_0$, we obtain

$$\lambda_0 = \lambda_s\sqrt{\frac{1+\beta}{1-\beta}}$$

which gives $\boxed{\lambda_0 = 687 \text{ nm}}$.

(c) What is the source moved around the Earth?

- Then in this case, $\theta = \pi/2$, and plutting into the formula

$$\lambda_0 = \lambda_s\frac{1}{\sqrt{1-\beta^2}}$$

which gives $\boxed{\lambda_0 = 549 \text{ nm}}$.

8. **Problem 8 - 2.56** The formula we need to consider is that for relativistic Doppler shift

$$\frac{1}{\lambda_0} = \frac{1}{\lambda_s}\frac{\sqrt{1-\beta^2}}{1+\beta\cos\theta}.$$

- Approaching the light ($\theta = \pi$), the observed wavelength is $\lambda_1 = 540$ nm.
- Recding from the light ($\theta = 0$), the observed wavelength is $\lambda_2 = 650$ nm.
- We must solve for the bike's speed and for the source wavelength (perceived at rest).
- When the bike is approaching, we have $\frac{1}{\lambda_1} = \frac{1}{\lambda_s}\sqrt{\frac{1+\beta}{1-\beta}}$. Isolating for the source wavelenght, we have $\lambda_s = \lambda_1\sqrt{\frac{1+\beta}{1-\beta}}$.
- When the bike is receding, we have $\frac{1}{\lambda_2} = \frac{1}{\lambda_s}\sqrt{\frac{1-\beta}{1+\beta}}$. (Note where the minus sign is!!!!) Isolating for the source wavelength again, we have $\lambda_s = \lambda_2\sqrt{\frac{1-\beta}{1+\beta}}$.
- Now we just have a system of equations with two unknowns and two equations. Equating the two expressions for $\lambda_s$ after a little bit of mathematical tap dancing, we find that

$$\beta = \frac{1-\frac{\lambda_1}{\lambda_2}}{1+\frac{\lambda_1}{\lambda_2}}$$

which when values are plugged in, gives $\boxed{\beta = 0.092}$. Then, using any of the above expressions for the source wavelength, we get $\boxed{\lambda_s = 592 \text{ nm}}$.