PHY293 Problem Set 5, Part 1

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1. Harris 2.70

The formula for relativistic momentum, total energy and kinetic energy are give by equations (2-22), (2-24) and (2-26) in the text book. Assuming $m_p = 1.67 \times 10^{-27}\, kg$ and the velocity of proton is in x direction ($u_x = 0.8\, c$),

$$\gamma = \frac{1}{\sqrt{1 - 0.8^2}} = \frac{5}{3}$$

$$p_x = \gamma m_p u_x = 6.68 \times 10^{-19}, p_y = p_z = 0 \quad \text{kg m/s}$$

$$E = \gamma m_p c^2 = 2.51 \times 10^{-10}\, J$$

$$KE = (\text{total energy}) - (\text{energy at rest}) = (\gamma - 1) mc^2 = 1.01 \times 10^{-10} \quad \text{J}$$

2. Harris 2.76

a) $\gamma_0.6 = 5/4$ and $\gamma_0.8 = 5/3$. Total relativistic momentum before the collision is given by

$$p_{tot}(before) = \sum p^i_{initial} = (5/4)(16)(0.6\, c) + (5/3)(9)(-0.8\, c) = 0$$

Total momentum after the collision is given by

$$p_{tot}(after) = \sum p^i_{final} = (5/4)(16)(-0.6\, c) + (5/3)(9)(0.8\, c) = 0$$

Total momentum is zero before and after the collision and is therefore conserved.
b) For the particle of mass 9 before the collision the velocity in the new frame (S') will be zero, since the frame moves at the same velocity as this particle. For the other three particles, we need to use the equation for relativistic velocity transformation which is given by

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

The velocity of S' with respect to S is $v = 0.6\, c$. For particle of mass 16 before the collision, we have $u_x = -0.6\, c$ and

$$u'_x = \frac{-0.8c - 0.6c}{1 - (-0.8)(0.6)} = -0.9459\, c$$

Similarly for the particle with mass 16 after the collision, we have

$$u'_x = \frac{-0.6c - 0.6c}{1 - (-0.6)(0.6)} = -0.8823\,c$$

And for the particle of mass 9 after the collision, the transformed velocity is given by

$$u'_x = \frac{0.8c - 0.6c}{1 - (0.8)(0.6)} = 0.3846\,c$$

c) Similar to part a, we need to calculate the total momentum before and after the collision using the transformed velocity values that are obtained in part b. The total initial momentum is given by

$$p'_{tot}(before) = \sum p'^{i}_{initial} = 0 + \gamma_{0.946}(9)(-0.946c) = -26.2$$

$$p'_{tot}(after) = \sum p'^{i}_{final} = \gamma_{0.882}(16)(-0.882c) + \gamma_{0.385}(9)(0.385c) = -26.2$$

So the value of the total momentum depends on the reference frame but it is conserved in all of the inertial reference frames.

3. Harris 2.81

Intensity is defined as power transferred per unit area, ie. $I = P/A$. We know that

$$P = \frac{dE}{dt} = IA$$

and also that

$$E = mc^2$$

Therefore,

$$\frac{dE}{dt} = \frac{dm}{dt}c^2$$

We want to find the rate at which mass is being converted to energy, that is $\frac{dm}{dt}$
So we have

$$\frac{dm}{dt} = \frac{1}{c^2}\frac{dE}{dt} = \frac{1}{c^2}IA$$

The intensity of sun's radiant energy on earth is sun's power distributed over a large sphere whose radius is equal to the distance between earth and sun, so $A = 4\pi d^2$. The rate of mass to energy conversion is therefore given by

$$\frac{dm}{dt} = \frac{4\pi d^2 I}{c^2} = \frac{4\pi(1.5\times10^{11})^2(1.5\times10^3)}{(3\times10^8)^2} = 4.71\times10^9\ kg/s$$

PHY293 Problem Set 5, Part 2

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Harris 2.87

a

The proton accelerates to a kinetic energy of 500MeV.

$$(\gamma - 1)m_p c^2 = 500\text{MeV}$$
$$\gamma = 1 + \frac{500\text{MeV}}{938\text{MeV}} = 1.53$$
$$u = 0.76c$$

b

Classically, the kinetic energy is given by:

$$\frac{1}{2}m_p v^2 = 500\text{MeV}$$
$$\frac{v^2}{c^2} = \frac{2 \times 500\text{MeV}}{938\text{MeV}}$$
$$v = 1.03c$$

Already this can't happen.

Quadrupling the potential would double the speed to $2.06c$. This really can't happen

c

The actual speed is:

$$\begin{aligned}(\gamma - 1)m_p c^2 &= 2000\text{MeV} \\ \gamma = 1 + \frac{2000\text{MeV}}{938\text{MeV}} &= 3.13 \\ u &= 0.95c\end{aligned}$$

Better.

Harris 2.93

a

In Experiment A the total kinetic energy is: $(\gamma_{0.9} - 1)m_0c^2$.
$\gamma_{0.9} = 2.29$

b

In Experiment B the total kinetic energy is: $2 \times (\gamma_u - 1)m_0c^2$.
If the total kinetic energies before the collision in both experiments are the same:

$$\begin{aligned}2 \times (\gamma_u - 1)m_0c^2 &= (\gamma_{0.9} - 1)m_0c^2 \\ \gamma_u = \frac{\gamma_{0.9} + 1}{2} &= 1.65 \\ u &= 0.79c\end{aligned}$$

c

Experiment A

The total energy before the collision is $(\gamma_{0.9} + 1)m_0c^2$. After the collision it is $\gamma_A m_A c^2$. Equating and rearranging gives: $\frac{m_A}{m_0} = \frac{\gamma_{0.9}+1}{\gamma_A}$.

The momentum before the collision is: $\gamma_{0.9}m_0(0.9c)$. After the collision it is $\gamma_A m_A u_A$. Equating and rearranging gives: $\frac{m_A}{m_0} = \frac{\gamma_{0.9}(0.9c)}{\gamma_A u_A}$.

From energy and momentum conservation we have:

$$\frac{\gamma_{0.9}+1}{\gamma_A} = \frac{\gamma_{0.9}(0.9c)}{\gamma_A u_A}$$
$$u_A = \frac{\gamma_{0.9}(0.9c)}{\gamma_{0.9}+1}$$
$$u_A = 0.63c$$
$$\gamma_A = 1.29$$
$$m_A = \frac{\gamma_{0.9}(0.9c)}{\gamma_A u_A} m_0 = 2.54m_0$$

Experiment B

After the collision in Experiment B, the kinetic energy is 0, and we expect that the mass is greater than in Experiment A. Equating the total energy before and after the collision gives:

$$2 \times \gamma_u m_0 c^2 = m_B c^2$$
$$m_B = 2 \times \gamma_u m_0 = 3.29m_0$$

Since the initial kinetic energies were equal and the final mass is greater in Experiment B, more of the initial kinetic energy was converted to mass.

Harris 2.94

a

From energy conservation:

$$\gamma_{K^0} m_{K^0} c^2 = \gamma_{0.9} m_\pi c^2 + \gamma_{0.8} m_\pi c^2$$
$$\gamma_{K^0} = \frac{(\gamma_{0.9}+\gamma_{0.8})m_\pi}{m_{K^0}}$$
$$\gamma_{K^0} = \frac{(1.67+2.29)*2.49}{8.87} = 1.11$$
$$u_{K^0} = 0.43c$$

b

We can define the directions of the pions by the angles of θ_+ and θ_- relative to the x-axis. From momentum conservation:

$$\begin{aligned}
p_x: \quad && \gamma_{K^0} m_{K^0} u_{K^0} &= \gamma_{0.9} m_\pi (0.9c) \cos\theta_+ + \gamma_{0.8} m_\pi (0.8c) \cos\theta_- \\
&& \gamma_{K^0} m_{K^0} u_{K^0} - \gamma_{0.9} m_\pi (0.9c) \cos\theta_+ &= \gamma_{0.8} m_\pi (0.8c) \cos\theta_- \\
p_y: \quad && 0 &= \gamma_{0.9} m_\pi (0.9c) \sin\theta_+ + \gamma_{0.8} m_\pi (0.8c) \sin\theta_- \\
&& -\gamma_{0.9} m_\pi (0.9c) \sin\theta_+ &= \gamma_{0.8} m_\pi (0.8c) \sin\theta_-
\end{aligned}$$

Squaring and summing the two rearranged momentum conservation equations gives:

$$\begin{aligned}
(\gamma_{K^0} m_{K^0} u_{K^0})^2 - 2(\gamma_{K^0} m_{K^0} u_{K^0})(\gamma_{0.9} m_\pi (0.9c) \cos\theta_+) + (\gamma_{0.9} m_\pi (0.9c))^2 &= (\gamma_{0.8} m_\pi (0.8c))^2 \\
\cos\theta_+ &= \frac{(\gamma_{K^0} m_{K^0} u_{K^0})^2 + (\gamma_{0.9} m_\pi (0.9c))^2 - (\gamma_{0.8} m_\pi (0.8c))^2}{2(\gamma_{K^0} m_{K^0} u_{K^0})(\gamma_{0.9} m_\pi (0.9c))} \\
\theta_+ &= 40.2^o \\
\sin\theta_- &= -\frac{\gamma_{0.9} m_\pi (0.9c) \sin\theta_+}{\gamma_{0.8} m_\pi (0.8c)} \\
\theta_- &= -83.9^o
\end{aligned}$$