PHY357 – Lecture 8

Quark composition of hadrons

Hadron magnetic moments

Hadron masses
Three types of stable quark configurations established:
- Baryons: $qqq$ has baryon number +1 (eg. proton, neutron, ...)
- Mesons: $qar{q}$ has baryon number 0 (eg. pions, kaons, ...)
- Anti-Baryons: $qar{q}ar{q}$ has baryon number -1 (eg. anti-proton, ...)

Baryon number conservation means:
- Baryon and anti-baryon can annihilate into mesons
- Lightest baryon (proton) will be stable
- Lightest meson(s) can only decay through EM or Weak process

\[ \pi^0 \rightarrow \gamma + \gamma \quad \pi^+ \rightarrow \mu^+ + \nu_\mu \]
Meson Multiplets (J=0)

- Saw last time how we could use iso-spin lowering operator to relate proton to neutron
- Can do the same thing for the pion multiplet (Parity even)

\[ \pi^+(u\bar{d}) : |1 \ 1\rangle = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \]

\[ \pi^0(u\bar{u}, d\bar{d}) : |1 \ 0\rangle = \sqrt{\begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}} + \sqrt{\begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \]

\[ \pi^- (\bar{u}d) : |1 \ -1\rangle = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \]

- Procedure would suggest that a state like

\[ \eta = (u\bar{u}, d\bar{d}) : |0 \ 0\rangle = \sqrt{\begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}} - \sqrt{\begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \]

- Actually exists and is isospin singlet, parity odd

\[ m = 135 \text{ MeV} \]
\[ m = 139 \text{ MeV} \]
\[ m = 135 \text{ MeV} \]
\[ m = 549 \text{ MeV} \]
The Strange ($s$) quark

- First saw $K$ mesons in accelerators along with pions (early 1950s)
- They became known as ‘strange’ mesons due to very long lifetimes
  - $\tau_{\pi^0} \sim 10^{-16}$ s, $\tau_K \sim 10^{-8}$ s
- Now understood because they include an $s$ quark that cannot decay strongly or electromagnetically
- In terms of isospin it is a singlet with $I=0$, $I_3 = 0$
  \[ s = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \]

- $K$ mesons contain only one $u$ (or $d$) quark and one $s$ quark
- Isospin doublets with:
  - $K^+(us)$: \[ \begin{pmatrix} 1 \\ 2 \end{pmatrix} |0,0\rangle \]
  - $K^-(\bar{u}s)$: \[ \begin{pmatrix} 1 \\ 2 \end{pmatrix} |0,0\rangle \]
  - $K^0(d\bar{s})$: \[ \begin{pmatrix} 1 \\ 2 \end{pmatrix} |0,0\rangle \]
  - $\bar{K}^0(\bar{d}s)$: \[ \begin{pmatrix} 1 \\ 2 \end{pmatrix} |0,0\rangle \]

Strangeness = +1 \quad Strangeness = -1
Define “Hypercharge” as number of non-u/d quarks

- \( Y = (\text{Baryon}) + S + C + T + B' \)
- Can draw all the light mesons in a single diagram

\( \eta' \) meson has \((uu + dd + ss)\) quark composition
The Meson Octet (u,d,s)

\[ S = 1 \quad S = 0 \quad S = -1 \]

- **Isospin doublet**
  - \( K^0 \) and \( K^+ \)
  - \( Q = -1 \) and \( Q = +1 \)

- **Isospin triplet**
  - \( \pi^- \) and \( \eta \)
  - \( Q = 0 \)

- **Isospin singlet**
  - \( \pi^0 \)
  - \( Q = 0 \)
Lowest Mass Baryons (J=1/2)

- Can do the same trick for the lowest mass (J=1/2) Baryons
  - A single diagram with different numbers of u/d/s quarks
Include Charm quarks (J=3/2)

- Charm becomes the vertical height in a pyramid
- NB. These are all J=3/2 combinations of quarks
- I=3/2 Δ resonances (uuu, uud, udd, ddd)
- One additional sss state, the Ω⁻ baryon
- Discovered shortly after quark spectroscopy was proposed – convincing evidence of quarks
Hadron Decays

- Proton is absolutely stable – lightest combination of 3 quarks

- Most other hadrons decay either:
  - Electromagnetically: If change of quark number is 0
    - Eg. $\pi^0 \rightarrow \gamma + \gamma$ ($\tau = 10^{-16}$ s)
  - Weakly: If one of the quark numbers changes by +/- 1
    - Eg. $K^+ \rightarrow \mu^+ + \nu_\mu$ ($\tau = 10^{-8}$ s)
    - $\Lambda \rightarrow p + \pi^-$ ($\tau = 10^{-10}$ s)

- To first order $\tau_{\text{strong}} \sim 10^{-22}$-$10^{-24}$s, $\tau_{\text{EM}} \sim 10^{-16}$-$10^{-21}$s, $\tau_{\text{weak}} \sim 10^{-7}$-$10^{-13}$s

- After that energy released/# of possible final states can
  - Decrease $\tau$ (higher Q, more states)
  - Increase $\tau$ (lower Q, fewer states)
    - Eg. Neutron decay ($Q < 1$ MeV, $\tau \sim 1000$s) – very slow
Hadron Magnetic Moments

- Quark model also predicts hadron magnetic moments
  - Only 3 free parameters ($\mu_N, m_u=m_d=m_s$)
  - Can then fit all low-mass baryon moments to better than 10%

- Based on the magnetic moment of a spin-1/2 object
  - Eg. Proton has 1 nuclear magneton: $\mu_N = e \hbar / (2 M_p)$

$$\mu_u = \frac{2M_p}{3m_u} \mu_N; \quad \mu_d = -\frac{M_p}{3m_d} \mu_N; \quad \mu_s = -\frac{M_p}{3m_s} \mu_N$$

- Have measured these for the eight lowest lying baryon states

- In the $\Lambda(uds)$ $ud$-quark are in spin-0 sub-state (no contribution)
  - So $\mu_\Lambda = \mu_s$
  - Very good agreement with observations (one way to fix $m_s$)
Magentic Moments Baryon Octet

- Baryon wavefunction constrains $|aa \rangle$ pair in $|aab \rangle$ state

- Leads to very simple prediction:
  \[
  \mu_B = \frac{4}{3} \mu_a - \frac{1}{3} \mu_b
  \]

Table 3.5  Magnetic moments of the $\frac{1}{2}^+$ baryon octet as predicted by the constituent quark model, compared with experiment in units of $\mu_N$, the nuclear magneton. These have been obtained using $m = 0.344 \text{ GeV}/c^2$ and $m_s = 0.539 \text{ GeV}/c^2$. Errors on the nucleon moments are of order $10^{-7}$.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Moment</th>
<th>Prediction</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(938)</td>
<td>$\frac{4}{3} \mu_u - \frac{1}{3} \mu_d$</td>
<td>2.73</td>
<td>2.793</td>
</tr>
<tr>
<td>n(940)</td>
<td>$\frac{4}{3} \mu_d - \frac{1}{3} \mu_u$</td>
<td>-1.82</td>
<td>-1.913</td>
</tr>
<tr>
<td>$\Lambda(1116)$</td>
<td>$\mu_s$</td>
<td>-0.58</td>
<td>-0.613 $\pm$ 0.004</td>
</tr>
<tr>
<td>$\Sigma^+(1189)$</td>
<td>$\frac{4}{3} \mu_u - \frac{1}{3} \mu_s$</td>
<td>2.62</td>
<td>2.458 $\pm$ 0.010</td>
</tr>
<tr>
<td>$\Sigma^-(1197)$</td>
<td>$\frac{4}{3} \mu_d - \frac{1}{3} \mu_s$</td>
<td>-1.02</td>
<td>-1.160 $\pm$ 0.025</td>
</tr>
<tr>
<td>$\Xi^+(1315)$</td>
<td>$\frac{4}{3} \mu_s - \frac{1}{3} \mu_u$</td>
<td>-1.38</td>
<td>-1.250 $\pm$ 0.014</td>
</tr>
<tr>
<td>$\Xi^-(1321)$</td>
<td>$\frac{4}{3} \mu_s - \frac{1}{3} \mu_d$</td>
<td>-0.47</td>
<td>-0.651 $\pm$ 0.003</td>
</tr>
</tbody>
</table>

$m_{u,d} = 344 \text{ MeV}$

$m_s = 539 \text{ MeV}$
To first order just depends on mass of the constituent quarks

- EM effects of like-charge pairs results in small corrections
- Leads to the prediction
  \[ m_{\Xi} - m_{\Sigma} = m_{\Sigma} - m_p = m_\Lambda - m_p = m_s - m_{u,d} \]
  
  Gives mass difference \( m_s - m_{u,d} \approx 150-200 \text{ MeV} \)
  
  Namely \( m_{u,d} = 300+ \text{ MeV} \) and \( m_s \approx 500 \text{ MeV} \)

- \( \Delta(uuu, uud, udd, ddd) \) resonances have very different mass from proton and neutron
  - Must be some spin dependent effects
Spin $\frac{1}{2}$ particles with magnetic moments have interaction energy:

$$\Delta E = \mu_1 \mu_2 / r^3$$

- Magnetic moment is given by $\mu = e/m \cdot S$
- Radial dependence averages over the relative wavefunction
- Leads to hyperfine splitting in paired electron orbitals (atomic physics)

$$\Delta E \sim S_1 \cdot S_2 / m_1 m_2$$

Same thing with strong force leading to mass corrections

- Can’t calculate absolute strength of strong splitting but just write
  $M(\text{meson}) = m_1 + m_2 + \Delta M$
- Where
  $$\Delta M \sim S_1 \cdot S_2 / m_1 m_2$$
- Two possibilities for $S_1 \cdot S_2$: $-\frac{3}{4} \hbar^2 (J=0)$ or $\frac{1}{4} \hbar^2 (J=1)$
- Leading to $\Delta M = -\frac{3}{4}a/m_1 m_2$ (J=0) and $\Delta M = \frac{1}{4} a/m_1 m_2$ (J=1)
Meson Predictions

\[ m_{u,d} = 308 \text{ MeV} \quad \text{and} \quad m_s = 480 \text{ MeV} \]
Baryon Mass Predictions

- Baryons are more complicated
- Include three combinations of spin pairings (among 3 quarks)
- Add up the mass splitting contributions from all three

Table 3.7  Baryon masses (in GeV/c^2) in the constituent quark model compared with experimental values. These have been obtained using m = 0.364 GeV/c^2, m_s = 0.537 GeV/c^2 and b = 0.0261 (GeV/c^2)^2.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass</th>
<th>Prediction</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>3m - \frac{3b}{4m^2}</td>
<td>0.94</td>
<td>0.939</td>
</tr>
<tr>
<td>Λ</td>
<td>2m + m_s - \frac{3b}{4} \left( \frac{1}{m_s^2} \right)</td>
<td>1.12</td>
<td>1.116</td>
</tr>
<tr>
<td>Σ</td>
<td>2m + m_s + \frac{b}{4} \left( \frac{1}{m_s^2} - \frac{4}{mm_s} \right)</td>
<td>1.18</td>
<td>1.193</td>
</tr>
<tr>
<td>Ξ</td>
<td>m + 2m_s + \frac{b}{4} \left( \frac{1}{m_s^2} - \frac{4}{mm_s} \right)</td>
<td>1.33</td>
<td>1.318</td>
</tr>
<tr>
<td>Δ</td>
<td>3m + \frac{3b}{4m^2}</td>
<td>1.23</td>
<td>1.232</td>
</tr>
<tr>
<td>Σ'</td>
<td>2m + m_s + \frac{b}{4} \left( \frac{1}{m_s^2} + \frac{2}{mm_s} \right)</td>
<td>1.38</td>
<td>1.385</td>
</tr>
<tr>
<td>Ξ'</td>
<td>m + 2m_s + \frac{b}{4} \left( \frac{2}{mm_s} + \frac{1}{m_s^2} \right)</td>
<td>1.53</td>
<td>1.533</td>
</tr>
<tr>
<td>Ω</td>
<td>3m_s + \frac{3b}{4m_s^2}</td>
<td>1.68</td>
<td>1.673</td>
</tr>
</tbody>
</table>

\[ m_{u,d} = 364 \text{ MeV} \]
\[ m_s = 537 \text{ MeV} \]
Summary

- Relatively simple set of rules for building hadrons from quarks
  - Can group hadrons into families
  - Based on isospin and number of 'other' quarks

- Can predict magnetic moments and masses
  - With relatively naïve assumptions about quark wavefunctions inside hadron bound states
  - After we get beyond $m_u = m_d$ and $m_s$ the corrections we discussed become less important
    - $m_c \sim 1.5$ GeV
    - $m_b \sim 4.8$ GeV
    - $m_t = 172.38$ GeV