Strong inhomogeneities
and non-Fermi liquids in
randomly depleted Kondo lattices
Strong inhomogeneities and non-Fermi liquids in randomly depleted Kondo lattices

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1. **Depleted Kondo lattice systems**
   From a single Kondo impurity to the Kondo lattice
   Experiments

2. **Strong-coupling limit**

3. **Large-\(N\) formalism and results**
   Strong inhomogeneities and non-FL
   Kondo-disorder phenomenology
Magnetic impurities in metals: Kondo effect

In the low-\(T\) limit, the impurity moment will be screened.

\[
\chi_{\text{imp}}(T \to 0) \to \text{const}
\]

\[
S_{\text{imp}}(T \to 0) \to 0
\]

\[
H_{\text{imp,1}} = J \sum_{kk'} c_{k\sigma} \sigma_{\sigma\sigma'} c_{k'\sigma'} = J \vec{S} \cdot \vec{S}_0
\]

Low-energy physics of Kondo model is determined by single scale

\[
T_K \sim D \exp(-D/J).
\]
Kondo lattice model

\[ H_{KLM} = \sum_{i,j \in \Lambda} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{i \in \Lambda} J S_i \cdot s_i \]

Kasuya (1956); Doniach (1977)
Single impurity & Kondo lattice

![Graph showing entropy per spin vs. logarithm of temperature with a peak at $T_K$.]
Single impurity & Kondo lattice

![Graph of Entropy per spin vs. ln T (left)](image)

![Graph of Resistivity vs. ln T (right)](image)
Single impurity & Kondo lattice

Entropy per spin

Resistivity

Protracted screening

Fermi liquid coherence

$T_{\text{coh}}$ $T_{\text{K}}$

$\ln T$

$\ln T$

$T_{\text{coh}}$ $T_{\text{K}}$
Energy scales of Kondo lattice

Single-impurity Kondo temperature \( T_K \)
Fermi-liquid coherence temperature \( T_{coh} \)

Usually \( T_{coh} \ll T_K \)

Are \( T_{coh} \) and \( T_K \) two different scales?
(Do they have a different \( J_K \) dependence?)
Energy scales of Kondo lattice

Single-impurity Kondo temperature $T_K$
Fermi-liquid coherence temperature $T_{coh}$

Usually $T_{coh} \ll T_K$

Slave-boson mean-field approximation:

$T_{coh} = c \, T_K$

where $c$ depends on conduction band only
Doniach's phase diagram for the Kondo lattice model

\[ H = \sum_{i<j} t_{ij} c_{i\sigma}^+ c_{j\sigma} + \sum_i J_K \bar{S}_i \cdot \vec{\sigma}_{\sigma\sigma'} c_{i\sigma'} \]

Two competing effects (at least):
* Kondo screening by conduction electrons \( T_K^{(1)} \sim D \exp(-D/J), D = \text{bandwidth} \)
* Magnetic ordering due to inter-impurity interaction \( I_{\text{RKKY}} \sim J_K^2/D \)
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\[ T_N \]
Static magnetic order

\[ T^* \]
Heavy Fermi liquid
(Fermi surface obeys Luttinger's theorem)

\[ T_K^{(1)} / I \]

SDW
FL?

H = \( t_{ij} c_{i\sigma}^+ c_{j\sigma} + \sum_i J_K \vec{S}_i \cdot \vec{\sigma}_{\sigma \sigma'} c_{i\sigma'} \)
Depleted Kondo lattices
Kondo lattice model

\[ H_{\text{KLM}} = \sum_{i,j \in \Lambda} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{i \in \Lambda} J S_i \cdot s_i \]

Kasuya (1956); Doniach (1977)
Depleted Kondo lattice model

\[ H_{KLM} = \sum_{i,j \in \Lambda} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{i \in \Lambda_{dep}} J S_i \cdot s_i \]
Kondo impurity model

\[ H_{KLM} = \sum_{i,j \in \Lambda} t_{ij} c_i^{\dagger} c_j \sigma + \sum_{i \in \Lambda_{dep}} J S_i \cdot s_i \]
Experiments: Depleted Kondo lattices

\[ H_{KLM} = \sum_{i,j \in \Lambda} t_{i,j} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i \in \Lambda_{dep}} J S_i \cdot s_i \]

Substituational doping, e.g., Ce → La
in compounds like CeCoIn\textsubscript{5}, CePb\textsubscript{3}, CeAl\textsubscript{2}, ....

Local-moment concentration \( n_f \) tunable between
\( n_f = 1 \) (dense lattice limit) and
\( n_f = 0 \) (single impurity limit)
Experiment: $\text{Ce}_x\text{La}_{1-x}\text{Au}_6$

Lin et al. (1987)
Experiment: Ce_{1-x}La_xPb_3

Single-impurity behavior between $n_f=0$ and $n_f=0.8$ (!)
Experiment: Ce$_{1-x}$La$_x$CoIn$_5$

Nakatsuji et al., PRL 89, 106402 (2002)
How are single-impurity Fermi liquid and heavy lattice Fermi liquid connected?
Strong-coupling limit
Strong coupling limit: $J/t \rightarrow \infty$

$n_c \lesssim 1$

$n_f = 1$: Kondo lattice, Fermi liquid of $(1-n_c)$ holes
Strong coupling limit: \( J/t \rightarrow \infty \)

\( n_c \lesssim 1 \)

\( n_f = 1: \) Kondo lattice, Fermi liquid of \((1-n_c)\) holes

\( n_f \rightarrow 0: \) Single impurity, Fermi liquid of \((n_c - n_f)\) particles
Strong coupling limit: $J/t \to \infty$

$n_c \lesssim 1$:

- $n_f = 1$: Kondo lattice, Fermi liquid of $(1 - n_c)$ holes
- $n_f \to 0$: Single impurity, Fermi liquid of $(n_c - n_f)$ particles
- $n_c = n_f$: Gapped insulator!
Large-$N$ formalism and results
Mean-field theory

\[ H_{KLM} = \sum_{i,j \in \Lambda} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{i \in \Lambda_{dep}} J S_i \cdot s_i \]

\[ S_i = f_{i\sigma}^\dagger \sigma_{\sigma'\sigma'} f_{i\sigma'}, \quad f_{i\sigma}^\dagger f_{i\sigma} = Q = N/2 \]

\[ H_{MF} = H_{\text{cond}} + b_i (c_{i\sigma}^\dagger f_{i\sigma} + \text{h.c.}) + \mu_i f_{i}^\dagger f_{i} \]

\[ b_i = J \langle c_{i}^\dagger f_{i} \rangle \quad \langle f_{i}^\dagger f_{i} \rangle = 1/2 \]

Read/Newns (1984); Millis/Lee (1987); Coleman (1987); Auerbach/Levin (1986)
Mean-field theory

\[ H_{KLM} = \sum_{i,j \in \Lambda} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{i \in \Lambda_{dep}} J S_i \cdot s_i \]

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Read/Newns (1984); Millis/Lee (1987)
Coleman (1987); Auerbach/Levin (1986)
Mean-field theory: Dense Kondo lattice

\[ H_{\text{MF}} = H_{\text{cond}} + b_i (c_i^{\dagger} f_i \sigma + \text{h.c.}) + \mu_i f_i^{\dagger} f_i \]

\[ b_i = \mathcal{J} \langle c_i^{\dagger} f_i \rangle \quad \langle f_i^{\dagger} f_i \rangle = 1/2 \]
Mean-field theory: Condensation at $T_K$

Define energy scale: $T^* = \frac{\langle |b_r(T = 0)|^2 \rangle}{D}$

$T^* = T_K$ for SI

$T^* = T_{\text{coh}}$ for KLM

Summary of mean-field results

(a)

$T^*/D$ vs $n_f$

- $n_c = 0.8$
- $n_c = 0.6$
- $n_c = 0.4$
- $n_c = 0.2$
- $n_c = 0.1$

Kaul/Vojta, cond-mat/0603002
Summary of mean-field results

Distribution of $b^2$ at $n_f = 0.5$

Sharp crossover around $n_f = n_c$

Kaul/Vojta, cond-mat/0603002
Summary of mean-field results

Kaul/Vojta, cond-mat/0603002
Local observables at fixed $n_c = 0.8$ and varying $n_f$

- $b_i, n_f = 0.05$
- LDOS, $n_f = 0.05$
- $\chi_{\text{loc}}, n_f = 0.05$
Local observables at fixed $n_c = 0.8$ and varying $n_f$

- $b_i, n_f = 0.05$
- LDOS, $n_f = 0.05$
- $\chi_{\text{loc}}, n_f = 0.05$

- $b_i, n_f = 0.95$
- LDOS, $n_f = 0.95$
- $\chi_{\text{loc}}, n_f = 0.95$
Isolated impurities

a) $b_i, n_f = 0.05$

LDOS, $n_f = 0.05$

$\chi_{\text{loc}}, n_f = 0.05$

b) $b_i, n_f = 0.3$

LDOS, $n_f = 0.3$

$\chi_{\text{loc}}, n_f = 0.3$

c) $b_i, n_f = 0.5$

LDOS, $n_f = 0.5$

$\chi_{\text{loc}}, n_f = 0.5$

d) $b_i, n_f = 0.95$

LDOS, $n_f = 0.95$

$\chi_{\text{loc}}, n_f = 0.95$

Isolated vacancies
Local DOS at fixed $n_f = 0.2$ and varying $n_c$

a) $b_i, n_c = 0.8$

b) LDOS, $n_c = 0.1$

c) LDOS, $n_c = 0.2$

d) LDOS, $n_c = 0.4$

e) LDOS, $n_c = 0.6$

f) LDOS, $n_c = 0.8$
Mean-field wavefunctions: Localization near $n_f = n_c$

$$1/\text{IPR}_n = \sum_i |\psi_n(r_i)|^4$$

Mean-field results for $n_f = 1, 0.9, 0.4, 0.25$. 

$n_c = 0.4$, 20X20, J/t = 1.25

- ipr c-elec
- ipr f-spin
- ipr full

$n_f > n_c$

$n_f = n_c$

$n_f < n_c$
Temperature dependence

Disorder is "self-generated" below $T_K$!
Relation to Kondo-disorder models

Disorder-Driven Non-Fermi-Liquid Behavior in Kondo Alloys

E. Miranda and V. Dobrosavljević
National High Magnetic Field Laboratory, Florida State University, 1800 E. Paul Dirac Drive, Tallahassee, Florida 32306
G. Kotliar

Random $J_K$ or random conduction band DOS $\rho$
$\rightarrow$ distribution of $(\rho J_K)$
$\rightarrow$ broad distribution of local $T_K$
$\rightarrow$ apparent non-Fermi-liquid behavior

Kondo disorder: Dobrosavljevic/Kirkpatrick/Kotliar (1993)
Summary

1. Depleted Kondo lattice: From Kondo Impurity to Heavy FL
2. Strongly inhomogeneous low-$T$ phases around $n_f = n_c$
3. NFL behavior (similar to Kondo disorder scenario)
4. Experiments on $\text{Ce}_{1-x}\text{La}_x\text{CoIn}_5$ & $\text{Ce}_{1-x}\text{La}_x\text{Pb}_3$
Mean-field theory: Spectrum

$n_c = 0.4$, 20X20, $J/t = 1.25$

![Graph showing MF Eigenvalues vs Eigenstate Index for different $n_f$ values.](image)
Argument for the Fermi surface volume of the FL phase

Single ion Kondo effect implies $J_K \to \infty$ at low energies

Fermi liquid of $S=1/2$ holes with hard-core repulsion

Fermi surface volume $= -(\text{density of holes}) \mod 2$

$= -(1 - n_c) = (1 + n_c) \mod 2$